

1. 10 pts. Determine the region in the xy -plane for which the differential equation $(9 - y^2)y' = x^2$ must have a unique solution whose graph passes through the point (x_0, y_0) in the region.
2. 10 pts. Suppose water is leaking from a tank through a circular hole of area A at its bottom. When water leaks through this hole, friction and contraction of the stream near the hole reduce the volume of water leaving the tank per second to $cA\sqrt{2gh}$, where $0 < c < 1$ is an empirical constant, g is the acceleration due to gravity, and h is the height of the water. Determine a differential equation for the height h of the water at time t in a $10 \text{ ft} \times 10 \text{ ft} \times 10 \text{ ft}$ cubical tank. The radius of the hole at the bottom is 2 inches, and $g = 32 \text{ ft/s}^2$.
3. 10 pts. Solve the initial-value problem by separation of variables:

$$x^2 \frac{dy}{dx} = y - xy, \quad y(-1) = -1.$$

4. 10 pts. Solve by separation of variables:

$$\sin 3x + (2y \cos^3 3x)y' = 0.$$

5. 10 pts. Solve the linear equation:

$$\frac{dP}{dt} + 2tP = P + 4t - 2.$$

6. 10 pts. Solve the initial value problem:

$$y' + 4xy = x^3 e^{x^2}, \quad y(0) = -1.$$

7. 10 pts. Solve the exact equation with given initial condition:

$$e^x + y + (2 + x + ye^y)y' = 0, \quad y(0) = 1.$$

8. 10 pts. Solve the differential equation by using an appropriate substitution:

$$\frac{dy}{dx} = \tan^2(x + y).$$

9. 10 pts. Solve the Bernoulli equation:

$$y' = y(xy^3 - 1).$$

A couple trigonometric identities: $\sin(2\theta) = 2 \sin \theta \cos \theta, \quad \cos(2\theta) = 2 \cos^2 \theta - 1.$