Math 250
Summer 2015
Exam 4

Name:

1. $\boxed{\text{10 pts.}}$ Use the definition of the Laplace transform to find $\mathcal{L}[f]$ for

$$f(t) = \begin{cases} 1, & t \in [0, 2) \\ t, & t \in [2, \infty) \end{cases}$$

2. $\boxed{\text{10 pts.}}$ Use the table provided to find the Laplace transform of $f(t) = (2t-1)^3$.

3. $\boxed{\text{10 pts.}}$ Find the inverse Laplace transform:

$$\mathcal{L}^{-1}\left[\frac{s+1}{s^2-4s}\right]$$

4. $\boxed{\text{15 pts.}}$ Use the method of Laplace transforms to solve the initial-value problem

$$y' - y = 2\cos 5t, \quad y(0) = 0.$$

5. $\boxed{\text{5 pts. each}}$ Find either $f(t)$ or $F(s)$, as indicated.

(a) $\mathcal{L}^{-1}\left[\dfrac{5s}{(s-2)^2}\right](t)$.

(b) $\mathcal{L}[(3t+1)u(t-1)](s)$.

(c) $\mathcal{L}^{-1}\left[\dfrac{e^{-\pi s}}{s^2+4}\right](t)$.

6. $\boxed{\text{15 pts.}}$ Use the Laplace transform to solve the initial-value problem

$$y' + 2y = f(t), \quad y(0) = 0,$$

where

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & t \ge 1. \end{cases}$$

7. $\boxed{\text{15 pts.}}$ Use the Laplace transform to solve the integral equation

$$f(t) = te^t + \int_0^t \tau f(t-\tau)\, d\tau.$$

8. $\boxed{\text{15 pts.}}$ Use the Laplace transform to solve the initial-value problem

$$y'' + 2y' = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 1.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s > 0$
$t \cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s > a$
$e^{at}t^n,\ n = 0, 1, \ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s > a$
$u(t-a),\ a \geq 0$	$\dfrac{e^{-as}}{s}$	$s > 0$
$\delta(t-a),\ a \geq 0$	e^{-as}	$s > 0$
$(f * g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x), \quad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$
$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$