

1. 15 pts. A thermometer is taken from inside a house to the great outdoors where the air temperature is  $5^\circ$  F. After 1 minute the thermometer reads  $55^\circ$  F, and after 5 minutes it reads  $30^\circ$  F. What is the temperature in the house?
2. 15 pts. A large tank is partially filled with 400 liters of water in which 5 kilograms of sugar is dissolved. Water containing 0.05 kg of sugar per liter is pumped into the tank at a rate of 20 L/min. The well-mixed solution is meanwhile pumped out at a slower rate of 15 L/min. Find the number of kilograms of sugar in the tank after 30 minutes.
3. 10 pts. Using either the Wronskian determinant or the definition of linear independence, determine whether the functions  $f(x) = x$ ,  $g(x) = x - 1$ , and  $h(x) = x + 3$  are linearly independent on  $(-\infty, \infty)$ .
4. 10 pts. Given that  $y = c_1 + c_2x^2$  is the general solution to  $xy'' - y' = 0$  on  $(-\infty, \infty)$ , find a solution to the boundary value problem

$$xy'' - y' = 0, \quad y(0) = 1, \quad y'(1) = 6.$$

5. 10 pts. Given that  $y_1(x) = x \sin(\ln x)$  is a solution to

$$x^2y'' - xy' + 2y = 0,$$

use reduction of order to find a second solution  $y_2(x)$ .

6. 10 pts. Find the general solution to  $2y'' + 2y' + y = 0$ .
7. 10 pts. Find the general solution to  $y''' - 6y'' + 12y' - 8y = 0$ .
8. 10 pts. Find a homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 + c_2x + c_3e^{8x}.$$

9. Use the Method of Undetermined Coefficients and the Superposition Principle in doing the following.
  - (a) 15 pts. Find a particular solution to

$$y'' + y' + 4y = 2 \cosh t,$$

where  $\cosh t = \frac{1}{2}(e^t + e^{-t})$ , and then find a general solution.

(b) 15 pts. Find the solution to the initial value problem

$$y'' + 2y' + 5y = 4e^{-t} \cos 2t, \quad y(0) = 1, \quad y'(0) = 0.$$

**A couple trigonometric identities:**  $\sin(2t) = 2 \sin t \cos t$ ,  $\cos(2t) = 2 \cos^2 t - 1$ .