

1. 10 pts. Determine for which values of m the function $\varphi(x) = e^{mx}$ is a solution to

$$2y'' + 7y' - 4y = 0.$$

2. 10 pts. Given that $y = \sin x$ is an explicit solution to the differential equation $y' = \sqrt{1 - y^2}$, find an interval I of definition.

3. 10 pts. Determine the region in the xy -plane for which the differential equation $(1 + y^3)y' = x^2$ must have a unique solution whose graph passes through the point (x_0, y_0) in the region.

4. 10 pts. A drug is infused into a patient's bloodstream at a constant rate of r grams per second. Simultaneously, the drug is removed at a rate proportional to the amount $x(t)$ of the drug present at time t . Determine a differential equation for the amount $x(t)$.

5. 10 pts. Find the general solution to

$$\frac{1}{y}y' + ye^{\cos x} \sin x = 0.$$

6. 10 pts. Solve the initial value problem

$$\frac{1}{2}y' = \sqrt{y+1} \cos x, \quad y(\pi) = 0.$$

7. 10 pts. Find the general solution to

$$xy' + 3(y + x^2) = \frac{\sin x}{x}.$$

8. 10 pts. Solve the initial value problem

$$t^2 \frac{dx}{dt} + 3tx = t^4 \ln t + 1, \quad x(1) = 0.$$

9. 10 pts. Solve the exact equation

$$(x - y^3 + y^2 \sin x) = (3xy^2 + 2y \cos x)y'$$

10. 10 pts. Solve the homogeneous equation $x^2 + y^2 + 2xyy' = 0$.

11. 10 pts. Solve the Bernoulli equation

$$y' = y(xy^3 - 1)$$