MATH 250
SUMMER 2014
EXAM 4

NAME:

1. 20 pts. Solve the initial value problem

$$y'' + 4y = \sin(t)u(t - 2\pi), \quad y(0) = 1, \quad y'(0) = 0$$

using the Method of Laplace Transforms.

2. 20 pts. Solve the initial value problem

$$y'' + 2y' = \delta(t - 1), \quad y(0) = 0, \quad y'(0) = 1.$$

using the Method of Laplace Transforms.

3. 10 pts. Find the inverse Laplace transform of

$$\frac{6}{(s^2 + 9)^2}$$

using the Convolution Theorem.

4. 15 pts. Determine the first four nonzero terms in the Taylor polynomial approximation of the solution to the initial value problem

$$y'' - t^3 y' + ty^2 = 0, \quad y(0) = -1, \quad y'(0) = 1.$$

5. 15 pts. Find the first four nonzero terms in a power series expansion about $t = 0$ for a general solution to $y' - y = 0$.

6. 20 pts. Find a power series expansion about the ordinary point $t = 0$ for the general solution to the differential equation

$$y'' + t^2 y' + ty = 0.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$$
$$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$$
$$\mathcal{L}[t^nf(t)](s)=(-1)^nF^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$$

$$\sin^2 x=\tfrac{1}{2}(1-\cos 2x),\quad \cos^2 x=\tfrac{1}{2}(1+\cos 2x)$$
$$\sin x\cos y=\tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y=\tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y=\tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$