Math 250
Summer 2014 Name:
Exam 3

1. A 1/8-kg object is attached to a spring with stiffness 16 N/m. The damping constant for the system is 2 N-sec/m. If the object is moved 3/4 m to the left of equilibrium (compressing the spring) and given an initial leftward velocity of 2 m/sec, determine the following.

   (a) [10 pts.] The equation of motion of the object.

   (b) [10 pts.] The object's maximum displacement to the left.

   (c) [10 pts.] The quasiperiod and quasifrequency of the object's motion.

2. [15 pts.] Use the definition of Laplace transform to determine $\mathcal{L}[f](s)$, where

$$f(t) = \begin{cases} 0, & 0 \leq t < \pi/2 \\ \cos t, & t \geq \pi/2 \end{cases}$$

3. [10 pts.] Determine $\mathcal{L}[f](s)$ using the table provided, where $f(t) = (t+1)^3$.

4. [10 pts.] Determine $\mathcal{L}[t^2 - e^{-9t} + 8](s)$.

5. [10 pts.] Determine $\mathcal{L}[\sin 4t \cos 3t](s)$.

6. [15 pts.] Determine $\mathcal{L}^{-1}[F](t)$, given that

$$F(s) = \frac{s}{s^2 + 2s - 3}.$$

7. [20 pts.] Solve the initial value problem

$$y' + 4y = e^{-4t}, \quad y(0) = 2$$

using the Method of Laplace Transforms.

8. [20 pts.] Solve the initial value problem

$$y'' - 6y' + 9y = t, \quad y(0) = 0, \quad y'(0) = 1$$

using the Method of Laplace Transforms.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$t^{-1/2}$	$\dfrac{\sqrt{\pi}}{\sqrt{s}}$	$s>0$
$t^{n-1/2},\ n=1,2,\ldots$	$\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s>0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[f'''](s) = s^3\mathcal{L}[f](s) - s^2f(0) - sf'(0) - f''(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$
$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$