

1. 10 pts. Solve the initial-value problem, where the differential equation is a homogeneous equation.

$$xy^2 \frac{dy}{dx} = y^3 - x^3, \quad y(1) = 2.$$

2. 10 pts. Solve the Bernoulli equation

$$y' = y(xy^3 - 1).$$

3. 20 pts. A tank with a capacity of 500 liters originally contains 200 L of water with 30 kg of salt in solution. Water containing 0.3 kg of salt per liter begins entering the tank at a rate of 4 L/min, and the mixture is allowed to flow out of the tank at a rate of 2 L/min. Find the amount of salt in the tank at any time t before the tank is full. Also find the concentration, in kg/L, of salt in the tank at the instant when the tank is full.

4. 10 pts. Solve the initial value problem $2y'' + 7y' - 15y = 0$, $y(0) = -2$, $y'(0) = 4$.

5. 10 pts. Find the general solution to $9y'' - 12y' + 4y = 0$.

6. 10 pts. Find the general solution to $12y''' - 28y'' - 3y' + 7y = 0$.

7. 10 pts. Solve the initial value problem $y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 1$.

8. Use the Method of Undetermined Coefficients and the Superposition Principle in doing the following.

- (a) 15 pts. Find a particular solution to

$$y'' + y' + 4y = 2 \cosh t,$$

where $\cosh t = \frac{1}{2}(e^t + e^{-t})$, and then find a general solution.

- (b) 15 pts. Find the solution to the initial value problem

$$y'' + 2y' + 5y = 4e^{-t} \cos 2t, \quad y(0) = 1, \quad y'(0) = 0.$$

9. 15 pts. Use the Method of Variation of Parameters to find a particular solution to

$$y'' - 2y' + y = \frac{e^t}{1 + t^2},$$

and then find a general solution.