Math 250
Summer 2014 Name:
Exam 1

1. $\boxed{\text{10 pts.}}$ Determine for which values of m the function $\varphi(x) = x^m$ is a solution to

$$3x^2 \frac{d^2y}{dx^2} + 11x\frac{dy}{dx} - 3y = 0.$$

2. $\boxed{\text{10 pts.}}$ Determine whether the Existence-Uniqueness Theorem implies that the initial value problem

$$\frac{dy}{dx} - 4x^2 = \sqrt[3]{2-y}, \quad y(-1) = 2,$$

has a unique solution. If not, why not?

3. $\boxed{\text{10 pts.}}$ Find the general solution to

$$e^x y\frac{dy}{dx} = e^{-y} + e^{-2x-y}.$$

4. $\boxed{\text{10 pts.}}$ Solve the initial value problem

$$\frac{dx}{dt} = 4(x^2+1), \quad x(\pi/4) = 1.$$

5. $\boxed{\text{10 pts.}}$ Find the general solution to

$$xy' + 3(y + x^2) = \frac{\sin x}{x}.$$

6. $\boxed{\text{10 pts.}}$ Solve the initial value problem

$$t^2\frac{dx}{dt} + 3tx = t^4 \ln t + 1, \quad x(1) = 0.$$

7. $\boxed{\text{10 pts.}}$ Solve the exact equation

$$\frac{x}{y}y' + (1 + \ln y) = 0.$$

8. $\boxed{\text{15 pts.}}$ Solve $y + (2xy - e^{-2y})y' = 0$ by finding an integrating factor of the form

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N}dx\right) \quad \text{or} \quad \mu(y) = \exp\left(\int \frac{N_x - M_y}{M}dy\right).$$

9. $\boxed{\text{15 pts.}}$ Solve $(x^3y^2 - 2y^3) + x^4yy' = 0$ by finding an integrating factor of the form $x^m y^n$.

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$