

1. 20 pts. Solve the initial value problem

$$z'' + 3z' + 2z = e^{-3t}u(t-2), \quad z(0) = 2, \quad z'(0) = -3$$

using the Method of Laplace Transforms.

2. 20 pts. Solve the initial value problem

$$y'' + 4y = 2\delta(t - \pi) - \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 0.$$

using the Method of Laplace Transforms.

3. 10 pts. Solve the integral equation

$$y(t) + \int_0^t (t - \tau)^2 y(\tau) d\tau = t^3 + 3.$$

using the Convolution Theorem.

4. 15 pts. Determine the first three nonzero terms in the Taylor polynomial approximation of the solution to the initial value problem

$$y'' + ty = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

5. 15 pts. Find the first four nonzero terms in a power series expansion about $x_0 = 0$ for a general solution to $y' - y = 0$.

6. 20 pts. Find the first four nonzero terms in a power series expansion about $t_0 = 0$ for a solution to the initial value problem

$$y'' + 3ty' - y = 0, \quad y(0) = 2, \quad y'(0) = 0.$$

$f(t)$	$\mathcal{L}[f](s)$	$\text{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$	$s > 0$
$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$	$s > a$
$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$	$s > a$
$e^{at} t^n, n = 0, 1, \dots$	$\frac{n!}{(s - a)^{n+1}}$	$s > a$
$u(t - a), a \geq 0$	$\frac{e^{-as}}{s}$	$s > 0$
$\delta(t - a), a \geq 0$	e^{-as}	$s > 0$
$(f * g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}[f(t - a)u(t - a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$

$$\mathcal{L}[g(t)u(t - a)](s) = e^{-as}\mathcal{L}[g(t + a)](s)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$