MATH 250
SUMMER 2013
EXAM 3

NAME:

1. 15 pts. A 20-kg object is attached to a spring with stiffness 200 N/m. The damping constant for the system is 140 N-sec/m. If the object is pulled 0.25 m to the right of equilibrium and given an initial leftward velocity of 1 m/sec, what is the equation of motion of the object? When will the object first return to its equilibrium position?

2. 15 pts. An 8-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 1.96 m upon coming to rest at equilibrium. At time $t = 0$, an external force $F(t) = \cos 2t$ N is applied to the system. The damping constant for the system is 3 N-sec/m. Determine the steady-state solution for the system.

3. 15 pts. Use the definition of Laplace transform to determine $\mathcal{L}[f](s)$, where

$$f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

4. 10 pts. Determine $\mathcal{L}[f](s)$ using the table on the back, where $f(t) = t^2 e^{5t}$.

5. 10 pts. Determine using table: $\mathcal{L}[t^5 - 7e^{-3t} \sin 4t](s)$.

6. 10 pts. Determine using table: $\mathcal{L}[e^{8t} \cos^2 t](s)$.

7. 15 pts. Determine using table: $\mathcal{L}^{-1}[F](t)$, given that

$$F(s) = \frac{s + 11}{(s-1)(s+3)}.$$

8. 20 pts. Solve the initial value problem

$$y'' + 2y' + 2y = t^2 + 4t, \quad y(0) = 0, \quad y'(0) = -1$$

using the Method of Laplace Transforms.

9. 20 pts. Solve the initial value problem

$$y''' - y'' + y' - y = 0, \quad y(0) = 1, \quad y'(0) = 1, \quad y''(0) = 3$$

using the Method of Laplace Transforms.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s > 0$
$t \cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s > a$
$e^{at} t^n,\ n = 0, 1, \ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s > a$
$t^{-1/2}$	$\dfrac{\sqrt{\pi}}{\sqrt{s}}$	$s > 0$
$t^{n-1/2},\ n = 1, 2, \ldots$	$\dfrac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[f'''](s) = s^3\mathcal{L}[f](s) - s^2 f(0) - sf'(0) - f''(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$
$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$
$$\Pi_{a,b}(t) = u(t-a) - u(t-b) = \begin{cases} 0, & \text{if } t < a \\ 1, & \text{if } a \le t < b \\ 0, & \text{if } t \ge b \end{cases}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \cos y = \tfrac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$