Math 250
Summer 2013
Exam 2

Name:

1. $\boxed{\text{10 pts.}}$ Solve the homogeneous equation $x^2 + y^2 + 2xyy' = 0$.

2. $\boxed{\text{10 pts.}}$ Solve the Bernoulli equation
$$y' = \frac{2y}{x} - x^2y^2.$$

3. $\boxed{\text{20 pts.}}$ Starting at $t = 0$, fresh water is pumped at the rate of 3 L/min into a 60 L tank initially filled with brine. The resulting less-and-less salty mixture overflows at the same rate into a second 60 L tank that initially contained only pure water, and from there it eventually spills onto the ground. Assuming perfect mixing in both tanks, when will the water in the second tank taste saltiest?

4. $\boxed{\text{10 pts.}}$ Using the definition of linear independence, determine whether the functions $\varphi(t) = te^{2t}$, $\psi(t) = e^{2t}$, and $\xi(t) = e^t$ are linearly independent on $(-\infty, \infty)$.

5. $\boxed{\text{10 pts.}}$ Solve the initial value problem $2y'' + 7y' - 15y = 0$, $y(0) = -2$, $y'(0) = 4$.

6. $\boxed{\text{10 pts.}}$ Find the general solution to $9y'' - 12y' + 4y = 0$.

7. $\boxed{\text{10 pts.}}$ Find the general solution to $12y''' - 28y'' - 3y' + 7y = 0$.

8. $\boxed{\text{10 pts.}}$ Solve the initial value problem $y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 1$.

9. Use the Method of Undetermined Coefficients (see other side) and the Superposition Principle to do the following.
   (a) $\boxed{\text{10 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = 4t^2$.
   (b) $\boxed{\text{15 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = e^t \sin t$.
   (c) $\boxed{\text{6 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = e^t \sin t + 4t^2$.
   (d) $\boxed{\text{6 pts.}}$ Present the general solution to $y'' - 3y' + 2y = e^t \sin t + 4t^2$.

10. Use the Method of Variation of Parameters to do the following.
   (a) $\boxed{\text{15 pts.}}$ Find a particular solution to $y'' + 4y' + 4y = e^{-2t} \ln(t)$.
   (b) $\boxed{\text{6 pts.}}$ Present the general solution to $y'' + 4y' + 4y = e^{-2t} \ln(t)$.
   (c) $\boxed{\text{2 pts.}}$ Why can't the Method of Undetermined Coefficients be used to solve the equation?

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m, and let $y_p(t)$ denote a particular solution to $a_2y'' + a_1y' + a_0y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then

$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^{m} A_k t^k,$$

   where
   (a) $s = 0$ if α is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if α is a single root of $a_2r^2 + a_1r + a_0 = 0$
   (c) $s = 2$ if α is a double root of $a_2r^2 + a_1r + a_0 = 0$

2. If $f(t) = P_m(t)e^{\alpha t}\cos\beta t$ or $f(t) = P_m(t)e^{\alpha t}\sin\beta t$ for $\beta \neq 0$, then

$$y_p(t) = t^s e^{\alpha t}\cos\beta t \sum_{k=0}^{m} A_k t^k + t^s e^{\alpha t}\sin\beta t \sum_{k=0}^{m} B_k t^k,$$

   where
   (a) $s = 0$ if $\alpha + \beta i$ is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if $\alpha + \beta i$ is a root of $a_2r^2 + a_1r + a_0 = 0$