Math 250
Summer 2013 Name:
Exam 1

1. [5 pts.] Write a differential equation that fits the physical description: The rate of change in the temperature T of coffee at time t is proportional to the difference between the temperature M of the air at time t and the temperature of the coffee at time t.

2. [10 pts.] Determine whether the Existence-Uniqueness Theorem implies that the initial value problem

$$\frac{dy}{dx} = 3x - \sqrt[3]{y-1},\ y(2) = 1,$$

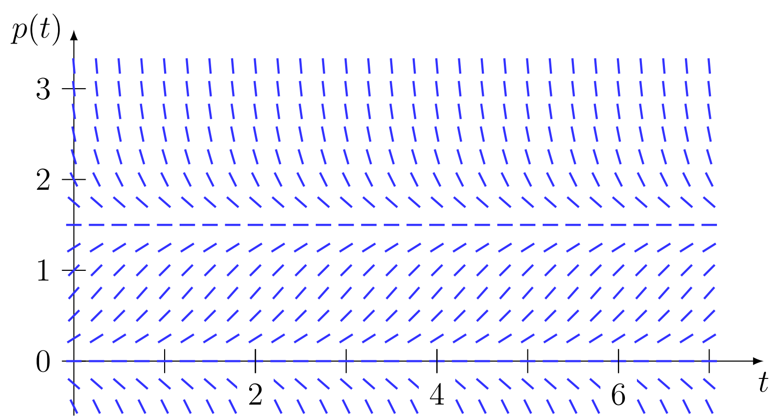
has a unique solution. If not, why not?

3. [10 pts.] Determine whether the function $\theta(t) = 2e^{3t} - e^{2t}$ is a solution to the differential equation

$$\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = -2e^{2t}.$$

4. [10 pts.] Determine for which values of m the function $\varphi(x) = e^{mx}$ is a solution to $2y''' + 9y'' - 5y' = 0$.

5. The logistic equation for the population p (in thousands) of a certain species is given by the equation $dp/dt = 3p - 2p^2$, which has the direction field below.



(a) [4 pts.] If the initial population is 3000 (i.e. $p(0) = 3$) what is the limiting population $\lim_{t\to\infty} p(t)$? Sketch the appropriate solution curve on the direction field.

(b) [4 pts.] If the initial population is 500 (i.e. $p(0) = 0.5$) what is the limiting population $\lim_{t\to\infty} p(t)$? Sketch the appropriate solution curve on the direction field.

(c) [3 pts.] Can a population of 2000 ever decline to 500?

6. [10 pts.] Use Euler's Method with step size $h = 0.1$ to approximate the solution to the initial value problem $y' = x - y^2$, $y(1) = 0$, at the points $x = 1.1, 1.2, 1.3, 1.4, 1.5$.

7. $\boxed{\text{10 pts.}}$ Find the general solution to $\dfrac{dy}{dx} = \dfrac{e^{x+y}}{y-1}$.

8. $\boxed{\text{10 pts.}}$ Solve the initial value problem $\frac{1}{2}y' = \sqrt{y+1}\cos x,\ \ y(\pi) = 0$.

9. $\boxed{\text{10 pts.}}$ Find the general solution to $xy' + 3(y + x^2) = \dfrac{\sin x}{x}$.

10. $\boxed{\text{10 pts.}}$ Solve the initial value problem $t^2\dfrac{dx}{dt} + 3tx = t^4 \ln t + 1,\ \ x(1) = 0$.

11. $\boxed{\text{10 pts.}}$ Solve the exact equation $(\cos x \cos y + 2x) - (\sin x \sin y + 2y)y' = 0$.

12. $\boxed{\text{15 pts.}}$ Solve $(3x^2 + y) + (2x^2 y - x)y' = 0$ by finding an integrating factor of the form

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N} dx\right) \quad \text{or} \quad \mu(y) = \exp\left(\int \frac{N_x - M_y}{M} dy\right).$$

13. $\boxed{\text{15 pts.}}$ Solve $(x^3y^2 - 2y^3) + x^4yy' = 0$ by finding an integrating factor of the form $x^m y^n$.

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$