MATH 250
SUMMER 2012
EXAM 4

NAME:

1. $\boxed{\text{20 pts.}}$ Solve the initial value problem $y'' - 7y' + 10y = 9\cos t + 7\sin t$, $y(0) = 5$, $y'(0) = -4$, using the Method of Laplace Transforms.

2. The current $I(t)$ in a circuit is governed by the initial value problem $I''(t) + 2I'(t) + 2I(t) = g(t)$, $I(0) = 10$, $I'(0) = 0$, where

$$g(t) = \begin{cases} 20, & 0 \le t \le 3\pi \\ 0, & 3\pi < t \le 4\pi \\ 20, & t > 4\pi \end{cases}$$

   (a) $\boxed{\text{10 pts.}}$ Express $g(t)$ in terms of window or step functions.

   (b) $\boxed{\text{10 pts.}}$ Find $\mathcal{L}[g(t)](s)$.

   (c) $\boxed{\text{20 pts.}}$ Solve the initial value problem by the Method of Laplace Transforms.

3. $\boxed{\text{10 pts. each}}$ Find the nth-order Taylor polynomial $P_n(x)$ with center x_0 for each function f.

   (a) $f(x) = \sin 2x$, with $n = 4$ and $x_0 = \pi/6$.

   (b) $f(x) = e^{x^2}$, with $n = 3$ and $x_0 = 0$.

4. $\boxed{\text{15 pts.}}$ Determine the interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)!}{n!}(x+2)^n$

5. $\boxed{\text{15 pts.}}$ Find the first four nonzero terms in a power series expansion about $x_0 = 0$ for a general solution to $y' - y = 0$.

6. $\boxed{\text{20 pts.}}$ Find the general solution to $y'' - x^2y' - xy = 0$ in the form of a power series about $x_0 = 0$. The answer should include a general formula for the coefficients.

7. $\boxed{\text{20 pts.}}$ Find the first four nonzero terms in a power series expansion about $x_0 = 2$ for a general solution to $x^2y'' - y' + y = 0$.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$t^{-1/2}$	$\dfrac{\sqrt{\pi}}{\sqrt{s}}$	$s>0$
$t^{n-1/2},\ n=1,2,\ldots$	$\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s>0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$
$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$
$$\Pi_{a,b}(t) = u(t-a) - u(t-b) = \begin{cases} 0, & \text{if } t<a \\ 1, & \text{if } a\le t<b \\ 0, & \text{if } t\ge b \end{cases}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$
$$\cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$