Math 250
Summer 2012 Name:
Exam 3

1. [10 pts.] A 1/4-kg mass is attached to a spring with stiffness 8 N/m. The damping constant for the system is 2 N-sec/m. If the mass is pushed 50 cm to the left of equilibrium and given a leftward velocity of 2 m/sec, when will the mass attain its maximum displacement to the left?

2. [20 pts.] A 2-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 20 cm upon coming to rest at equilibrium. At time $t = 0$ the mass is displaced 5 cm below the equilibrium position and released. At this same instant, an external force $F(t) = 0.3\cos(t)$ N is applied to the system. If the damping constant for the system is 5 N-sec/m, determine the equation of motion for the mass.

3. [10 pts.] Use the definition of Laplace transform to determine $\mathcal{L}[f](s)$, where

$$f(t) = \begin{cases} 1 - t, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

4. [10 pts.] Determine the Laplace transform of $f(t) = t^2 e^{5t}$.

5. [10 pts.] Use the Laplace transform table and linearity to determine $\mathcal{L}[t^5 - 7e^{-3t}\sin 4t](s)$.

6. [15 pts.] Determine $\mathcal{L}[e^{8t}\cos^2 t](s)$.

7. [15 pts.] Determine $\mathcal{L}^{-1}[F](t)$, given that $F(s) = \dfrac{7s^2 - 41s + 84}{(s-1)(s^2 - 4s + 13)}$.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	$s>a$
$t^{-1/2}$	$\frac{\sqrt{\pi}}{\sqrt{s}}$	$s>0$
$t^{n-1/2},\ n=1,2,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s>0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$
$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$
$$\Pi_{a,b}(t) = u(t-a) - u(t-b) = \begin{cases} 0, & \text{if } t<a \\ 1, & \text{if } a\le t<b \\ 0, & \text{if } t\ge b \end{cases}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$
$$\cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$