

MATH 250
SUMMER 2012
EXAM 2

NAME:

1. 10 pts. Solve the homogeneous equation $y' = \frac{y^2 + x\sqrt{x^2 + y^2}}{xy}$.

2. 10 pts. Solve the Bernoulli equation $y' - y = e^{2x}y^3$.

3. 20 pts. A swimming pool whose volume is 10,000 gallons contains water that is 0.01% chlorine. Starting at time $t = 0$, city water containing 0.001% chlorine is pumped into the pool at a rate of 5 gal/min. The pool water flows out at the same rate. What is the percentage of chlorine in the pool after 1 hour? When will the pool water be 0.002% chlorine?

4. 10 pts. Solve the initial value problem $y'' - 4y' - 5y = 0$, $y(-1) = 3$, $y'(-1) = 9$.

5. 10 pts. Find the general solution to $y''' - 6y'' - y' + 6y = 0$.

6. 10 pts. Find the general solution to $y'' - 2y' + 26y = 0$.

7. Consider the equation $y'' + 4y = 8 \sin 2t$.
 - (a) 10 pts. Find a particular solution to the equation.
 - (b) 5 pts. Find a general solution to the equation.

8. Consider the equation $2y'' + 3y' + y = t^2 + 3 \sin t$.
 - (a) 15 pts. Find a particular solution to the equation.
 - (b) 5 pts. Find a general solution to the equation.

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m , and let $y_p(t)$ denote a particular solution to $a_2y'' + a_1y' + a_0y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then

$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^m A_k t^k,$$

where

- (a) $s = 0$ if α is not a root of $a_2r^2 + a_1r + a_0 = 0$
- (b) $s = 1$ if α is a single root of $a_2r^2 + a_1r + a_0 = 0$
- (c) $s = 2$ if α is a double root of $a_2r^2 + a_1r + a_0 = 0$

2. If $f(t) = P_m(t)e^{\alpha t} \cos \beta t$ or $f(t) = P_m(t)e^{\alpha t} \sin \beta t$ for $\beta \neq 0$, then

$$y_p(t) = t^s e^{\alpha t} \cos \beta t \sum_{k=0}^m A_k t^k + t^s e^{\alpha t} \sin \beta t \sum_{k=0}^m B_k t^k,$$

where

- (a) $s = 0$ if $\alpha + \beta i$ is not a root of $a_2r^2 + a_1r + a_0 = 0$
- (b) $s = 1$ if $\alpha + \beta i$ is a root of $a_2r^2 + a_1r + a_0 = 0$