Math 250
Summer 2012 Name:
Exam 1

1. $\boxed{\text{5 pts.}}$ Write a differential equation that fits the physical description: The rate of change in the temperature T of coffee at time t is proportional to the difference between the temperature M of the air at time t and the temperature of the coffee at time t.

2. $\boxed{\text{5 pts.}}$ Classify the differential equation

$$\frac{d^2y}{dx^2} - 0.1(1-y^2)\frac{dy}{dx} + 9y = 0$$

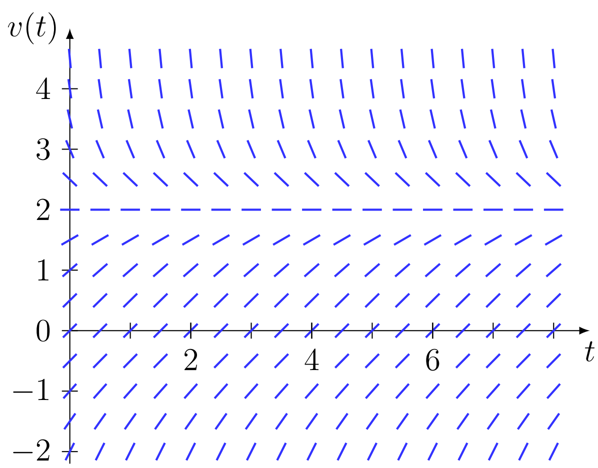
as an ordinary or partial differential equation, give the equation's order, and indicate the independent and dependent variables. If the equation is an ordinary differential equation, indicate whether the equation is linear or nonlinear.

3. $\boxed{\text{10 pts.}}$ Determine whether the function $\theta(t) = 2e^{3t} - e^{2t}$ is a solution to the differential equation

$$\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = -2e^{2t}.$$

4. $\boxed{\text{10 pts.}}$ Determine for which values of m the function $\varphi(x) = e^{mx}$ is a solution to $y''' + 3y'' + 2y' = 0$.

5. $\boxed{\text{10 pts.}}$ The velocity v at time t of an object falling in a viscous fluid is modeled by $v' = 1 - v^3/8$, with direction field given below. Sketch the solutions with initial conditions $v(0) = 0, 2, 4$. What velocity does the object approach as time increases? (This is known as the terminal velocity.)



6. $\boxed{\text{10 pts.}}$ Use Euler's Method with step size $h = 0.1$ to approximate the solution to the initial value problem $y' = x - y^2$, $y(1) = 0$, at the points $x = 1.1, 1.2, 1.3, 1.4, 1.5$.

7. $\boxed{\text{10 pts.}}$ Find the general solution to $\dfrac{dx}{dt} = \dfrac{t}{xe^{t+2x}}$.

8. $\boxed{\text{10 pts.}}$ Solve the initial value problem $\dfrac{1}{2}y' = \sqrt{y+1}\cos x,\ \ y(\pi) = 0$.

9. $\boxed{\text{10 pts.}}$ Find the general solution to $xy' + 3(y + x^2) = \dfrac{\sin x}{x}$.

10. $\boxed{\text{10 pts.}}$ Solve the initial value problem $t^2\dfrac{dx}{dt} + 3tx = t^4 \ln t + 1,\ \ x(1) = 0$.

11. $\boxed{\text{10 pts.}}$ Solve the exact equation $\left(2x + \dfrac{y}{1+x^2y^2}\right) + \left(\dfrac{x}{1+x^2y^2} - 2y\right) y' = 0$. Recall the integration formula $\displaystyle\int \frac{1}{a^2+u^2}du = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + c$.

12. $\boxed{\text{15 pts.}}$ Solve $(y^2 + 2xy) - x^2y' = 0$ by finding by an integrating factor $\mu(x) = \exp\left(\displaystyle\int \frac{M_y - N_x}{N}dx\right)$ or $\mu(y) = \exp\left(\displaystyle\int \frac{N_x - M_y}{M}dy\right)$.

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$