Math 250
Spring 2018
Exam 4

Name:

1. [10 pts. each] A 1/8-kg object is attached to a spring with stiffness 16 N/m. The damping constant for the system is 2 N-sec/m. If the object is moved 3/4 m to the left of equilibrium (compressing the spring) and given an initial leftward velocity of 2 m/sec, determine the following.

   (a) The equation of motion of the object.

   (b) The object's maximum displacement to the left.

2. [20 pts.] Use the power series method to solve the initial-value problem

$$y'' + xy' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

3. [15 pts.] Use the definition of the Laplace transform to find $\mathcal{L}[f(t)](s)$ for

$$f(t) = \begin{cases} e^{2t}, & 0 \le t < 3 \\ 1, & t \ge 3 \end{cases}$$

4. [5 pts. each] Use the table of Laplace transforms to find each.

   (a) $\mathcal{L}[t^5 - 7e^{-3t}\sin 4t](s)$

   (b) $\mathcal{L}[\sin t \cos 2t]$

5. [20 pts.] Solve using the Laplace transform method:

$$y'' - 2y' - y = e^{2t} - e^t, \quad y(0) = 1, \quad y'(0) = 3.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\dots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$

$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin x\cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$