MATH 250
SPRING 2018 NAME:
EXAM 2

1. [10 pts.] When a vertical beam of light passes through a transparent medium, the rate at which the intensity I decreases is proportional to $I(t)$, where t is the thickness of the medium in meters. In a certain lake the intensity 1.5 meters below the surface is 7% of the initial intensity I_0 of the incident beam. What is the intensity of the beam 2.5 meters below the surface?

2. [15 pts.] A small metal bar, whose initial temperature is 20°C, is dropped into a large container of boiling water. How long will it take the bar to reach 90°C if it is known that its temperature increased by 2° in the first second? How long will it take the bar to reach 98°C?

3. [15 pts.] A large tank is partially filled with 400 liters of water in which 4 kilograms of sugar is dissolved. Water containing 0.04 kg of sugar per liter is pumped into the tank at a rate of 18 L/min. The well-mixed solution is meanwhile pumped out at a slower rate of 15 L/min. Find the number of kilograms of sugar in the tank at time t.

4. [15 pts.] Show that the set of functions $\{1, x, \cos x, \sin x\}$ is a fundamental set of solutions to

$$y^{(4)} + y'' = 0$$

on the interval $(-\infty, \infty)$, then write out the general solution to the ODE.

5. [10 pts.] Given that $y_1(t) = t^2$ is a solution to

$$t^2y'' + 2ty' - 6y = 0,$$

use reduction of order to find a second solution $y_2(t)$.

6. [10 pts. each] Find the general solution to each.
   (a) $2y'' - 7y' + 3y = 0$
   (b) $y''' + 3y'' - 4y' - 12y = 0$

7. [15 pts.] Solve the initial-value problem:

$$y'' - 2y' + y = 0, \quad y(0) = 5, \quad y'(0) = 10.$$