1. 10 pts. Determine the points $\left(x_{0}, y_{0}\right)$ in the $x y$-plane for which the initial-value problem

$$
y^{\prime} \sqrt{6-y}=x y, \quad y\left(x_{0}\right)=y_{0}
$$

must have a unique solution.
2. 10 pts . The air resistance encountered by a falling body is proportional to the square of its instantaneous velocity. Find a differential equation for the velocity $v(t)$ of a falling body of mass $m$, where $t$ is time in seconds.
3. 10 pts . Solve the differential equation

$$
\frac{d N}{d t}+N=N t e^{t+2}
$$

4. Consider the initial-value problem

$$
\frac{d y}{d x}=\frac{3 x-1}{4 y}, \quad y(-2)=-1
$$

(a) 10 pts. Find the explicit solution to the initial-value problem.
(b) 5 pts. Find the solution's interval of validity by analytical means.
5. 10 pts . Solve the linear equation

$$
x y^{\prime}+y=x^{2}+1
$$

6. 10 pts . Solve the exact equation with given initial condition:

$$
e^{x}+y+\left(2+x+y e^{y}\right) y^{\prime}=0, \quad y(0)=1
$$

7. 10 pts. Solve the homogeneous differential equation:

$$
x+y e^{y / x}-x e^{y / x} y^{\prime}=0 .
$$

8. 10 pts . Solve the Bernoulli equation:

$$
x \frac{d y}{d x}+6 y=3 x y^{4 / 3}
$$

A couple trigonometric identities: $\quad \sin (2 \theta)=2 \sin \theta \cos \theta, \quad \cos (2 \theta)=2 \cos ^{2} \theta-1$.

