Math 250 Spring 2018 Exam 1

## NAME:

1. 10 pts. Determine the points  $(x_0, y_0)$  in the xy-plane for which the initial-value problem

$$y'\sqrt{6-y} = xy, \quad y(x_0) = y_0$$

must have a unique solution.

- 2. 10 pts. The air resistance encountered by a falling body is proportional to the square of its instantaneous velocity. Find a differential equation for the velocity v(t) of a falling body of mass m, where t is time in seconds.
- 3. 10 pts. Solve the differential equation

$$\frac{dN}{dt} + N = Nte^{t+2}.$$

4. Consider the initial-value problem

$$\frac{dy}{dx} = \frac{3x-1}{4y}, \quad y(-2) = -1.$$

(a) 10 pts. Find the explicit solution to the initial-value problem.

(b) 5 pts. Find the solution's interval of validity by analytical means.

5. 10 pts. Solve the linear equation

$$xy' + y = x^2 + 1.$$

6. 10 pts. Solve the exact equation with given initial condition:

$$e^{x} + y + (2 + x + ye^{y})y' = 0, \quad y(0) = 1.$$

7. 10 pts. Solve the homogeneous differential equation:

$$x + ye^{y/x} - xe^{y/x}y' = 0.$$

8. 10 pts. Solve the Bernoulli equation:

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}$$

A couple trigonometric identities:  $\sin(2\theta) = 2\sin\theta\cos\theta$ ,  $\cos(2\theta) = 2\cos^2\theta - 1$ .