

MATH 250  
SPRING 2017  
EXAM 2

NAME:

1. 10 pts. each Initially 100 mg of a radioactive isotope was present. After 6 hours the mass of the isotope had decreased by 3.4%.

  - (a) If the rate of decay of the isotope is proportional to the mass present at time  $t$ , find the mass remaining after 24 hours.
  - (b) When will 90% of the isotope be gone?
2. 15 pts. A dead body was found in a house where the temperature was a constant  $68^\circ$  F. At the time of discovery the core temperature of the body was determined to be  $83^\circ$  F. One hour later a second measurement showed the core temperature to be  $77^\circ$  F. Assume the core temperature at the time of death was  $98.6^\circ$  F. How much time elapsed between the time of death and the time the body was found?
3. 15 pts. A large tank is partially filled with 400 liters of water in which 4 kilograms of sugar is dissolved. Water containing 0.04 kg of sugar per liter is pumped into the tank at a rate of 18 L/min. The well-mixed solution is meanwhile pumped out at a slower rate of 15 L/min. Find the number of kilograms of sugar in the tank at time  $t$ .

4. 10 pts. Show that the functions  $x^3$  and  $x^4$  form a fundamental set of solutions to

$$x^2y'' - 6xy' + 12y = 0$$

on the interval  $(0, \infty)$ .

5. 10 pts. each Find the general solution to each.

(a)  $2y'' - 7y' + 3y = 0$

(b)  $y''' + 3y'' - 4y' - 12y = 0$

6. 10 pts. each Solve the initial-value problem:

$$y'' - 6y' + 25y = 0, \quad y(0) = 3, \quad y'(0) = 1.$$

7. 10 pts. Find a homogeneous linear differential equation with constant coefficients whose general solution is

$$y = c_1 + c_2e^{2x} \cos 5x + c_3e^{2x} \sin 5x.$$