MATH 250
SPRING 2018 NAME:
EXAM 1

1. [10 pts.] Determine the points (x_0, y_0) in the xy-plane for which the initial-value problem

$$y'\sqrt{6-y} = xy, \quad y(x_0) = y_0$$

must have a unique solution.

2. [10 pts.] The air resistance encountered by a falling body is proportional to the square of its instantaneous velocity. Find a differential equation for the velocity $v(t)$ of a falling body of mass m, where t is time in seconds.

3. [10 pts.] Solve the differential equation

$$\frac{dN}{dt} + N = Nte^{t+2}.$$

4. Consider the initial-value problem

$$\frac{dy}{dx} = \frac{3x-1}{4y}, \quad y(-2) = -1.$$

(a) [10 pts.] Find the explicit solution to the initial-value problem.

(b) [5 pts.] Find the solution's interval of validity by analytical means.

5. [10 pts.] Solve the linear equation

$$xy' + y = x^2 + 1.$$

6. [10 pts.] Solve the exact equation with given initial condition:

$$e^x + y + (2 + x + ye^y)y' = 0, \quad y(0) = 1.$$

7. [10 pts.] Solve the homogeneous differential equation:

$$x + ye^{y/x} - xe^{y/x}y' = 0.$$

8. [10 pts.] Solve the Bernoulli equation:

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}.$$

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$