MATH 250
SPRING 2016 NAME:
EXAM 4

1. [10 pts.] A 1-kilogram mass is attached to a spring whose constant is 16 N/m, and the entire system is then submerged in a liquid that imparts a damping force numerically equal to 10 times the instantaneous velocity. Determine the equation of motion if the mass is initially released from a point 1 meter below the equilibrium position with an upward velocity of 12 m/s.

2. [10 pts.] Rewrite the expression

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 5\sum_{n=0}^{\infty} c_n x^{n+2}$$

using a single power series whose general term involves x^n.

3. [15 pts.] Find a power series solution $y = \sum c_n x^n$ for $y' = xy$ by the power series method of §6.1.

4. [20 pts.] Use the power series method to solve the initial-value problem

$$y'' - 2xy' + 8y = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

5. [10 pts.] Use the definition of the Laplace transform to find $\mathcal{L}[f]$ for

$$f(t) = \begin{cases} 1, & t \in [0, 8) \\ t, & t \in [8, \infty) \end{cases}$$

6. [5 pts. each] Use the table provided to find the Laplace transform.
   (a) $f(t) = -2t^5$
   (b) $g(t) = (2t-1)^3$
   (c) $h(t) = e^t \sinh t$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
1	$\frac{1}{s}$	$s > 0$
t^n	$\frac{n!}{s^{n+1}}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$
$\sin bt$	$\frac{b}{s^2+b^2}$	$s > 0$
$\cos bt$	$\frac{s}{s^2+b^2}$	$s > 0$
$\sinh bt$	$\frac{b}{s^2-b^2}$	
$\cosh bt$	$\frac{s}{s^2-b^2}$	

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$