MATH 250
SPRING 2016
EXAM 3

NAME:

1. $\boxed{\text{10 pts.}}$ Given that $y_1(t) = t^2$ is a solution to

$$t^2 y'' + 2ty' - 6y = 0,$$

use reduction of order to find a second solution $y_2(t)$.

2. Consider the equation $y'' + 2y' = 2t + 5 - e^{-2t}$.

   (a) $\boxed{\text{15 pts.}}$ Use the Method of Undetermined Coefficients and the Superposition Principle to find a particular solution to the equation.

   (b) $\boxed{\text{5 pts.}}$ Give the general solution to the equation.

3. $\boxed{\text{15 pts.}}$ Use the Method of Undetermined Coefficients to solve the initial-value problem

$$y'' + 4y = -2, \quad y(\pi/8) = \tfrac{1}{2}, \quad y'(\pi/8) = 2.$$

4. $\boxed{\text{20 pts.}}$ Use the Method of Variation of Parameters to find a particular solution to

$$2y'' + 2y' + y = 4\sqrt{t},$$

and then find a general solution.

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m, and let $y_p(t)$ denote a particular solution to $a_n y^{(n)} + \cdots + a_1 y' + a_0 y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then

$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^{m} A_k t^k,$$

where $s = 0$ if α is not a root of the auxiliary equation, otherwise s equals the multiplicity of α as a root of the auxiliary equation.

2. If $f(t) = P_m(t)e^{\alpha t}\cos\beta t$ or $f(t) = P_m(t)e^{\alpha t}\sin\beta t$ for $\beta \neq 0$, then

$$y_p(t) = t^s e^{\alpha t}\cos\beta t \sum_{k=0}^{m} A_k t^k + t^s e^{\alpha t}\sin\beta t \sum_{k=0}^{m} B_k t^k,$$

where $s = 0$ if $\alpha + i\beta$ is not a root of the auxiliary equation, otherwise s equals the multiplicity of $\alpha + i\beta$ as a root of the auxiliary equation.

Method of Variation of Parameters (2nd-Order Case).

$$v_1(t) = \frac{1}{a_2}\int \frac{-y_2(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}\,dt \quad \text{and} \quad v_2(t) = \frac{1}{a_2}\int \frac{y_1(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}\,dt$$

Some Most Excellent Formulae.

1. $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$

2. $\displaystyle\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$

3. $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$

4. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$

5. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$

6. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$

7. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$