MATH 250
SPRING 2016 NAME:
EXAM 2

1. [15 pts.] A thermometer reading 70°F is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer notes that the thermometer reads 120°F after half a minute and 160°F after one minute. How hot is the oven?

2. [15 pts.] A large tank is partially filled with 400 liters of water in which 5 kilograms of sugar is dissolved. Water containing 0.05 kg of sugar per liter is pumped into the tank at a rate of 20 L/min. The well-mixed solution is meanwhile pumped out at a slower rate of 15 L/min. Find the number of kilograms of sugar in the tank after one hour.

3. [10 pts.] Using either the Wronskian determinant or the definition of linear independence, determine whether the functions $f(x) = x$, $g(x) = 6x - 1$, and $h(x) = 2x + 3$ are linearly independent on $(-\infty, \infty)$.

4. [10 pts.] Given that $y = c_1x^2 + c_2x^4 + 3$ is a two-parameter family of solutions to $x^2y'' - 5xy' + 8y = 24$ on $(-\infty, \infty)$, determine whether a member of the family exists that satisfies the boundary conditions $y(0) = 3$ and $y(1) = 0$.

5. [10 pts.] Find the general solution to $y'' - 10y' + 25y = 0$.

6. [10 pts.] Find the general solution to $y^{(4)} + y''' + y'' = 0$.

7. [10 pts.] Solve the initial value problem:

$$4y'' - 4y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 5.$$

A couple trigonometric identities: $\sin(2t) = 2\sin t \cos t$, $\cos(2t) = 2\cos^2 t - 1$.