MATH 250
SPRING 2016
EXAM 1

NAME:

1. $\boxed{\text{10 pts.}}$ Determine the region in the xy-plane for which the differential equation $(9 - y^2)y' = x^2$ must have a unique solution whose graph passes through the point (x_0, y_0) in the region.

2. $\boxed{\text{10 pts.}}$ Suppose water is leaking from a tank through a circular hole of area A_h at its bottom. When water leaks through a hole, friction and contraction of the stream near the hole reduce the volume of water leaving the tank per second to $cA_h\sqrt{2gh}$, where $0 < c < 1$ is an empirical constant, g is the acceleration due to gravity, and h is the height of the water. Determine a differential equation for the height h of the water at time t in a 10 ft $\times$ 10 ft $\times$ 10 ft cubical tank. The radius of the hole at the bottom is 2 inches, and $g = 32$ ft/s^2.

3. $\boxed{\text{10 pts.}}$ Solve the initial-value problem by separation of variables:

$$x^2\frac{dy}{dx} = y - xy, \quad y(-1) = -1.$$

4. $\boxed{\text{10 pts.}}$ Solve by separation of variables:

$$\sin 3x + (2y\cos^3 3x)y' = 0.$$

5. $\boxed{\text{10 pts.}}$ Solve the linear equation:

$$\frac{dP}{dt} + 2tP = P + 4t - 2.$$

6. $\boxed{\text{10 pts.}}$ Solve the initial value problem:

$$y' + 4xy = x^3e^{x^2}, \quad y(0) = -1.$$

7. $\boxed{\text{10 pts.}}$ Solve the exact equation with given initial condition:

$$e^x + y + (2 + x + ye^y)y' = 0, \quad y(0) = 1.$$

8. $\boxed{\text{10 pts.}}$ Solve the differential equation by using an appropriate substitution:

$$\frac{dy}{dx} = \tan^2(x + y).$$

9. $\boxed{\text{10 pts.}}$ Solve the Bernoulli equation:

$$y' = y(xy^3 - 1).$$

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$