

1. 5 pts. each Find either $f(t)$ or $F(s)$, as indicated.

(a) $\mathcal{L}^{-1}\left[\frac{5s}{(s-2)^2}\right](t).$

(b) $\mathcal{L}[(3t+1)u(t-1)](s).$

(c) $\mathcal{L}^{-1}\left[\frac{e^{-\pi s}}{s^2+4}\right](t).$

2. 15 pts. Use the Laplace transform to solve the initial-value problem

$$y' + 2y = f(t), \quad y(0) = 0,$$

where

$$f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 0, & t \geq 1. \end{cases}$$

3. 15 pts. Use the Laplace transform to solve the integral equation

$$f(t) = te^t + \int_0^t \tau f(t-\tau) d\tau.$$

4. 15 pts. Use the Laplace transform to solve the initial-value problem

$$y'' + 2y' = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 1.$$

$f(t)$	$\mathcal{L}[f](s)$	$\text{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$	$s > 0$
$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$	$s > a$
$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$	$s > a$
$e^{at} t^n, n = 0, 1, \dots$	$\frac{n!}{(s - a)^{n+1}}$	$s > a$
$u(t - a), a \geq 0$	$\frac{e^{-as}}{s}$	$s > 0$
$\delta(t - a), a \geq 0$	e^{-as}	$s > 0$
$(f * g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}[f(t - a)u(t - a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$

$$\mathcal{L}[g(t)u(t - a)](s) = e^{-as}\mathcal{L}[g(t + a)](s)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x), \quad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$