Math 250
Spring 2015 Name:
Exam 4

1. $\boxed{\text{10 pts.}}$ Rewrite the expression

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 5\sum_{n=0}^{\infty} c_n x^{n+2}$$

using a single power series whose general term involves x^n.

2. $\boxed{\text{15 pts.}}$ Find a power series solution $y = \sum c_n x^n$ for

$$(2x-4)y' + y = 0$$

by the power series method used in the homework.

3. $\boxed{\text{20 pts.}}$ Use the power series method to solve the initial-value problem

$$y'' - 2xy' + 8y = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

4. $\boxed{\text{10 pts.}}$ Use the definition of the Laplace transform to find $\mathcal{L}[f]$ for

$$f(t) = \begin{cases} 1, & t \in [0,2) \\ t, & t \in [2,\infty) \end{cases}$$

5. $\boxed{\text{10 pts.}}$ Use the table provided to find the Laplace transform of $f(t) = (2t-1)^3$.

6. $\boxed{\text{10 pts.}}$ Find the inverse Laplace transform:

$$\mathcal{L}^{-1}\left[\frac{s+1}{s^2-4s}\right]$$

7. $\boxed{\text{15 pts.}}$ Use the method of Laplace transforms to solve the initial-value problem

$$y' - y = 2\cos 5t, \quad y(0) = 0.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\dots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$$
$$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$$
$$\mathcal{L}[t^nf(t)](s)=(-1)^nF^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$$

$$\sin^2 x=\tfrac{1}{2}(1-\cos 2x),\quad \cos^2 x=\tfrac{1}{2}(1+\cos 2x)$$
$$\sin x\cos y=\tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y=\tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y=\tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$