Math 250
Spring 2015
Exam 3

Name:

1. Use the Method of Undetermined Coefficients and the Superposition Principle in doing the following.

   (a) [15 pts.] Find a particular solution to

   $$y'' + y' + 4y = 2\cosh t,$$

   where $\cosh t = \frac{1}{2}(e^t + e^{-t})$, and then find a general solution.

   (b) [15 pts.] Find the solution to the initial value problem

   $$y'' + 2y' + 5y = 4e^{-t}\cos 2t, \quad y(0) = 1, \quad y'(0) = 0.$$

2. [15 pts.] Use the Method of Variation of Parameters to find a particular solution to

   $$y'' - 2y' + y = \frac{e^t}{1+t^2},$$

   and then find a general solution.

3. An object weighing 64 pounds stretches a spring 0.32 foot. The mass is initially released from a point 8 inches above the equilibrium position with a downward velocity of 5 ft/s. Find the following.

   (a) [5 pts.] The equation of motion of the object.

   (b) [5 pts.] The position, velocity, and acceleration of the object at time $t = 3$ s.

   (c) [5 pts.] The amplitude and period of motion.

   (d) [5 pts.] The velocity at the times when the object passes through the equilibrium position.

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m, and let $y_p(t)$ denote a particular solution to $a_n y^{(n)} + \cdots + a_1 y' + a_0 y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then

$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^{m} A_k t^k,$$

   where $s = 0$ if $\alpha + i\beta$ is not a root of the auxiliary equation, otherwise s equals the multiplicity of $\alpha + i\beta$ as a root of the auxiliary equation.

2. If $f(t) = P_m(t)e^{\alpha t}\cos\beta t$ or $f(t) = P_m(t)e^{\alpha t}\sin\beta t$ for $\beta \neq 0$, then

$$y_p(t) = t^s e^{\alpha t}\cos\beta t \sum_{k=0}^{m} A_k t^k + t^s e^{\alpha t}\sin\beta t \sum_{k=0}^{m} B_k t^k,$$

   where $s = 0$ if $\alpha + i\beta$ is not a root of the auxiliary equation, otherwise s equals the multiplicity of $\alpha + i\beta$ as a root of the auxiliary equation.

Method of Variation of Parameters.

$$v_1(t) = \frac{1}{a_2}\int \frac{-y_2(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}dt \quad \text{and} \quad v_2(t) = \frac{1}{a_2}\int \frac{y_1(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}dt$$

Some Most Excellent Formulae.

1. $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
2. $\displaystyle\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
3. $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
4. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$
5. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
6. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
7. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$