MATH 250
SPRING 2014 NAME:
EXAM 4

1. 10 pts. Use the definition of Laplace transform to determine $\mathcal{L}[f](s)$, where

$$f(t) = \begin{cases} e^{2t}, & 0 \le t < 3 \\ 1, & t \ge 3 \end{cases}$$

2. 10 pts. Determine $\mathcal{L}[f](s)$ using the table on the back, where $f(t) = t^8 e^{-6t}$.

3. 10 pts. Determine using table: $\mathcal{L}[t^5 - 7e^{-3t}\sin 4t](s)$.

4. 10 pts. Determine using table: $\mathcal{L}[\sin 3t \cos 7t](s)$.

5. 10 pts. Determine using table: $\mathcal{L}^{-1}[F](t)$, given that

$$F(s) = \frac{s-4}{(s+3)^2}.$$

6. 10 pts. Solve for $Y(s)$, the Laplace transform of the solution $y(t)$ of the initial value problem

$$y'' - 2y' - y = e^{2t} - e^t, \quad y(0) = 1, \quad y'(0) = 3.$$

7. 20 pts. Solve the initial value problem

$$y'' - y = t - 2, \quad y(2) = 3, \quad y'(2) = 0$$

using the Method of Laplace Transforms.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$t^{-1/2}$	$\dfrac{\sqrt{\pi}}{\sqrt{s}}$	$s>0$
$t^{n-1/2},\ n=1,2,\ldots$	$\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s>0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$
$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$
$$\mathcal{L}[f'''](s) = s^3\mathcal{L}[f](s) - s^2f(0) - sf'(0) - f''(0)$$
$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$
$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$
$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$
$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$
$$\Pi_{a,b}(t) = u(t-a) - u(t-b) = \begin{cases} 0, & \text{if } t<a \\ 1, & \text{if } a\le t<b \\ 0, & \text{if } t\ge b \end{cases}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$
$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$