MATH 250
SPRING 2014 NAME:
EXAM 3

1. Use the Method of Undetermined Coefficients and the Superposition Principle in doing the following.

   (a) [15 pts.] Find a particular solution to

   $$y'' + y' + 4y = 2\cosh t,$$

   where $\cosh t = \frac{1}{2}(e^t + e^{-t})$, and then find a general solution.

   (b) [15 pts.] Find the solution to the initial value problem

   $$y'' + 2y' + 5y = 4e^{-t}\cos 2t, \quad y(0) = 1, \quad y'(0) = 0.$$

2. [15 pts.] Use the Method of Variation of Parameters to find a particular solution to

   $$y'' - 2y' + y = \frac{e^t}{1+t^2},$$

   and then find a general solution.

3. A 1/8-kg object is attached to a spring with stiffness 16 N/m. The damping constant for the system is 2 N-sec/m. If the object is moved 3/4 m to the left of equilibrium (compressing the spring) and given an initial leftward velocity of 2 m/sec, determine the following.

   (a) [10 pts.] The equation of motion of the object.

   (b) [10 pts.] The object's maximum displacement to the left.

   (c) [10 pts.] The quasiperiod and quasifrequency of the object's motion.

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m, and let $y_p(t)$ denote a particular solution to $a_2y'' + a_1y' + a_0y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then
$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^{m} A_k t^k,$$
where
   (a) $s = 0$ if α is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if α is a single root of $a_2r^2 + a_1r + a_0 = 0$
   (c) $s = 2$ if α is a double root of $a_2r^2 + a_1r + a_0 = 0$
2. If $f(t) = P_m(t)e^{\alpha t}\cos\beta t$ or $f(t) = P_m(t)e^{\alpha t}\sin\beta t$ for $\beta \neq 0$, then
$$y_p(t) = t^s e^{\alpha t}\cos\beta t \sum_{k=0}^{m} A_k t^k + t^s e^{\alpha t}\sin\beta t \sum_{k=0}^{m} B_k t^k,$$
where
   (a) $s = 0$ if $\alpha + \beta i$ is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if $\alpha + \beta i$ is a root of $a_2r^2 + a_1r + a_0 = 0$

Method of Variation of Parameters.

$$v_1(t) = \frac{1}{a_2}\int \frac{-y_2(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}dt \quad \text{and} \quad v_2(t) = \frac{1}{a_2}\int \frac{y_1(t)f(t)}{y_1(t)y_2'(t) - y_1'(t)y_2(t)}dt$$

Some Awesome Formulae.

1. $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
2. $\displaystyle\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
3. $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
4. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$
5. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
6. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
7. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$