MATH 250
SPRING 2014 NAME:
EXAM 2

1. [15 pts.] Solve $y + (2xy - e^{-2y})y' = 0$ by finding an integrating factor of the form

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N} dx\right) \quad \text{or} \quad \mu(y) = \exp\left(\int \frac{N_x - M_y}{M} dy\right).$$

2. [15 pts.] Solve $(x^3y^2 - 2y^3) + x^4yy' = 0$ by finding an integrating factor of the form $x^m y^n$.

3. [10 pts.] Solve the homogeneous equation $x^2 + y^2 + 2xyy' = 0$.

4. [10 pts.] Solve the Bernoulli equation

$$y' = \frac{2y}{x} - x^2y^2.$$

5. [20 pts.] A tank with a capacity of 500 liters originally contains 200 L of water with 30 kg of salt in solution. Water containing 0.3 kg of salt per liter begins entering the tank at a rate of 4 L/min, and the mixture is allowed to flow out of the tank at a rate of 2 L/min. Find the amount of salt in the tank at any time t before the tank is full. Also find the concentration, in kg/L, of salt in the tank at the instant when the tank is full.

6. [10 pts.] Solve the initial value problem $2y'' + 7y' - 15y = 0$, $y(0) = -2$, $y'(0) = 4$.

7. [10 pts.] Find the general solution to $9y'' - 12y' + 4y = 0$.

8. [10 pts.] Find the general solution to $12y''' - 28y'' - 3y' + 7y = 0$.

9. [10 pts.] Solve the initial value problem $y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 1$.

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = 2\cos^2\theta - 1$.