MATH 250
SPRING 2014
EXAM 1

NAME:

1. [10 pts.] Determine for which values of m the function $\varphi(x) = x^m$ is a solution to

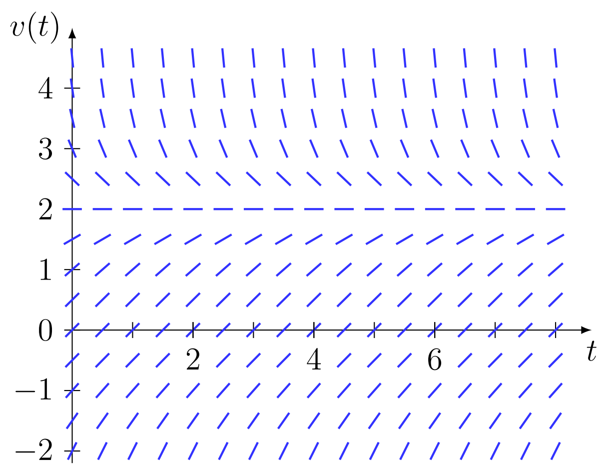
$$3x^2\frac{d^2y}{dx^2} + 11x\frac{dy}{dx} - 3y = 0.$$

2. [10 pts.] Determine whether the Existence-Uniqueness Theorem implies that the initial value problem

$$\frac{dy}{dx} - 4x^2 = \sqrt[3]{2-y}, \quad y(-1) = 2,$$

has a unique solution. If not, why not?

3. The velocity v at time t of an object falling in a viscous fluid is modeled by $v' = 1 - v^3/8$, with direction field given below.



(a) [5 pts.] Sketch the solutions with initial conditions $v(0) = 0, 2, 4$.

(b) [5 pts.] What velocity does the object approach as time increases?

4. [10 pts.] Use Euler's Method with step size $h = 0.2$ to approximate the solution to the initial value problem

$$y' = \frac{1}{x}(y^2 + y), \quad y(1) = 0$$

at the points $x = 1.2, 1.4, 1.6, 1.8$.

5. [10 pts.] Find the general solution to

$$\frac{1}{y}y' + ye^{\cos x}\sin x = 0.$$

6. [10 pts.] Solve the initial value problem

$$\frac{1}{2}y' = \sqrt{y+1}\cos x, \quad y(\pi) = 0.$$

7. [10 pts.] Find the general solution to

$$xy' + 3(y + x^2) = \frac{\sin x}{x}.$$

8. [10 pts.] Solve the initial value problem

$$t^2 \frac{dx}{dt} + 3tx = t^4 \ln t + 1, \quad x(1) = 0.$$

9. [10 pts.] Solve the exact equation

$$\frac{x}{y} y' + (1 + \ln y) = 0.$$

A couple trigonometric identities: $\sin(2\theta) = 2 \sin\theta \cos\theta, \quad \cos(2\theta) = 2\cos^2\theta - 1.$