MATH 250
SPRING 2013 NAME:
EXAM 5

1. [20 pts.] Solve the initial value problem

$$z'' + 3z' + 2z = e^{-3t}u(t-2), \quad z(0) = 2, \quad z'(0) = -3$$

using the Method of Laplace Transforms.

2. [20 pts.] Solve the initial value problem

$$y'' + 4y = 2\delta(t-\pi) - \delta(t-2\pi), \quad y(0) = 0, \quad y'(0) = 0.$$

using the Method of Laplace Transforms.

3. [10 pts.] Solve the integral equation

$$y(t) + \int_0^t (t-\tau)^2 y(\tau)\, d\tau = t^3 + 3.$$

using the Convolution Theorem.

4. [15 pts.] Determine the first three nonzero terms in the Taylor polynomial approximation of the solution to the initial value problem

$$y'' + ty = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

5. [15 pts.] Find the first four nonzero terms in a power series expansion about $x_0 = 0$ for a general solution to $y' - y = 0$.

6. [20 pts.] Find the first four nonzero terms in a power series expansion about $t_0 = 0$ for a solution to the initial value problem

$$y'' + 3ty' - y = 0, \quad y(0) = 2, \quad y'(0) = 0.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$

$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$

$\mathcal{L}[t^nf(t)](s)=(-1)^nF^{(n)}(s)$

$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x=\frac{1}{2}(1-\cos 2x), \quad \cos^2 x=\frac{1}{2}(1+\cos 2x)$

$\sin x\cos y=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$