

1. 15 pts. Use the definition of Laplace transform to determine $\mathcal{L}[f](s)$, where

$$f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

2. 10 pts. Determine $\mathcal{L}[f](s)$ using the table on the back, where $f(t) = t^2 e^{5t}$.

3. 10 pts. Determine using table: $\mathcal{L}[t^5 - 7e^{-3t} \sin 4t](s)$.

4. 10 pts. Determine using table: $\mathcal{L}[e^{8t} \cos^2 t](s)$.

5. 15 pts. Determine using table: $\mathcal{L}^{-1}[F](t)$, given that

$$F(s) = \frac{s + 11}{(s - 1)(s + 3)}.$$

6. 20 pts. Solve the initial value problem

$$y'' + 2y' + 2y = t^2 + 4t, \quad y(0) = 0, \quad y'(0) = -1$$

using the Method of Laplace Transforms.

7. 20 pts. Solve the initial value problem

$$y''' - y'' + y' - y = 0, \quad y(0) = 1, \quad y'(0) = 1, \quad y''(0) = 3$$

using the Method of Laplace Transforms.

$f(t)$	$\mathcal{L}[f](s)$	$\text{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$	$s > 0$
$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$	$s > a$
$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$	$s > a$
$e^{at} t^n, n = 0, 1, \dots$	$\frac{n!}{(s - a)^{n+1}}$	$s > a$
$t^{-1/2}$	$\frac{\sqrt{\pi}}{\sqrt{s}}$	$s > 0$
$t^{n-1/2}, n = 1, 2, \dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n - 1) \sqrt{\pi}}{2^n s^{n+1/2}}$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

$$\mathcal{L}[f'''](s) = s^3\mathcal{L}[f](s) - s^2f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}[f(t - a)u(t - a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$

$$\mathcal{L}[g(t)u(t - a)](s) = e^{-as}\mathcal{L}[g(t + a)](s)$$

$$\mathcal{L}[u(t - a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$

$$\Pi_{a,b}(t) = u(t - a) - u(t - b) = \begin{cases} 0, & \text{if } t < a \\ 1, & \text{if } a \leq t < b \\ 0, & \text{if } t \geq b \end{cases}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$