MATH 250
SPRING 2013 NAME:
EXAM 3

1. Use the Method of Undetermined Coefficients (see other side) and the Superposition Principle to do the following.
   (a) $\boxed{\text{10 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = 4t^2$.
   (b) $\boxed{\text{15 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = e^t \sin t$.
   (c) $\boxed{\text{6 pts.}}$ Find a particular solution to $y'' - 3y' + 2y = e^t \sin t + 4t^2$.
   (d) $\boxed{\text{6 pts.}}$ Present the general solution to $y'' - 3y' + 2y = e^t \sin t + 4t^2$.

2. Use the Method of Variation of Parameters to do the following.
   (a) $\boxed{\text{15 pts.}}$ Find a particular solution to $y'' - 2y' + y = t^{-1}e^t$.
   (b) $\boxed{\text{6 pts.}}$ Present the general solution to $y'' - 2y' + y = t^{-1}e^t$.
   (c) $\boxed{\text{2 pts.}}$ Why can't the Method of Undetermined Coefficients be used to solve the equation?

3. $\boxed{\text{15 pts.}}$ A 20-kg object is attached to a spring with stiffness 200 N/m. The damping constant for the system is 140 N-sec/m. If the object is pulled 0.25 m to the right of equilibrium and given an initial leftward velocity of 1 m/sec, what is the equation of motion of the object? When will the object first return to its equilibrium position?

4. $\boxed{\text{15 pts.}}$ An 8-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 1.96 m upon coming to rest at equilibrium. At time $t = 0$, an external force $F(t) = \cos 2t$ N is applied to the system. The damping constant for the system is 3 N-sec/m. Determine the steady-state solution for the system.

Method of Undetermined Coefficients. Let $P_m(t)$ be a nonzero polynomial of degree m, and let $y_p(t)$ denote a particular solution to $a_2y'' + a_1y' + a_0y = f(t)$.

1. If $f(t) = P_m(t)e^{\alpha t}$, then

$$y_p(t) = t^s e^{\alpha t} \sum_{k=0}^{m} A_k t^k,$$

   where
   (a) $s = 0$ if α is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if α is a single root of $a_2r^2 + a_1r + a_0 = 0$
   (c) $s = 2$ if α is a double root of $a_2r^2 + a_1r + a_0 = 0$

2. If $f(t) = P_m(t)e^{\alpha t}\cos\beta t$ or $f(t) = P_m(t)e^{\alpha t}\sin\beta t$ for $\beta \neq 0$, then

$$y_p(t) = t^s e^{\alpha t}\cos\beta t \sum_{k=0}^{m} A_k t^k + t^s e^{\alpha t}\sin\beta t \sum_{k=0}^{m} B_k t^k,$$

   where
   (a) $s = 0$ if $\alpha + \beta i$ is not a root of $a_2r^2 + a_1r + a_0 = 0$
   (b) $s = 1$ if $\alpha + \beta i$ is a root of $a_2r^2 + a_1r + a_0 = 0$