

1. 15 pts. Solve  $(3x^2 + y) + (2x^2y - x)y' = 0$  by finding an integrating factor of the form

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N} dx\right) \quad \text{or} \quad \mu(y) = \exp\left(\int \frac{N_x - M_y}{M} dy\right).$$

2. 15 pts. Solve  $(x^3y^2 - 2y^3) + x^4yy' = 0$  by finding an integrating factor of the form  $x^m y^n$ .

3. 10 pts. Solve the homogeneous equation  $y' = \frac{y(\ln y - \ln x + 1)}{x}$ .

4. 10 pts. Solve the Bernoulli equation  $y' + y = e^x y^{-2}$ .

5. 20 pts. Blood enters an organ at a rate of  $3 \text{ cm}^3/\text{sec}$  and leaves at the same rate. The organ has a liquid volume of  $125 \text{ cm}^3$ . If the concentration of a drug in the blood entering the organ is  $0.2 \text{ g/cm}^3$ , what is the concentration of the drug in the organ at time  $t$  if there was no trace of the drug initially? When will the concentration of the drug in the organ reach  $0.1 \text{ g/cm}^3$ ?

6. 10 pts. Using the definition of linear independence, determine whether the functions  $\varphi(t) = te^{2t}$  and  $\psi(t) = e^{2t}$  are linearly independent.

7. 10 pts. Solve the initial value problem  $y'' - 4y' - 5y = 0$ ,  $y(-1) = 3$ ,  $y'(-1) = 9$ .

8. 10 pts. Find the general solution to  $9y'' - 12y' + 4y = 0$ .

9. 10 pts. Find the general solution to  $12y''' - 28y'' - 3y' + 7y = 0$ .

10. 10 pts. Solve the initial value problem  $y'' + 9y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 1$ .