

1. 5 pts. Write a differential equation that fits the physical description: The rate of change in the temperature T of coffee at time t is proportional to the difference between the temperature M of the air at time t and the temperature of the coffee at time t .

2. 10 pts. Determine whether the Existence-Uniqueness Theorem implies that the initial value problem

$$\frac{dy}{dx} = 3x - \sqrt[3]{y-1}, \quad y(2) = 1,$$

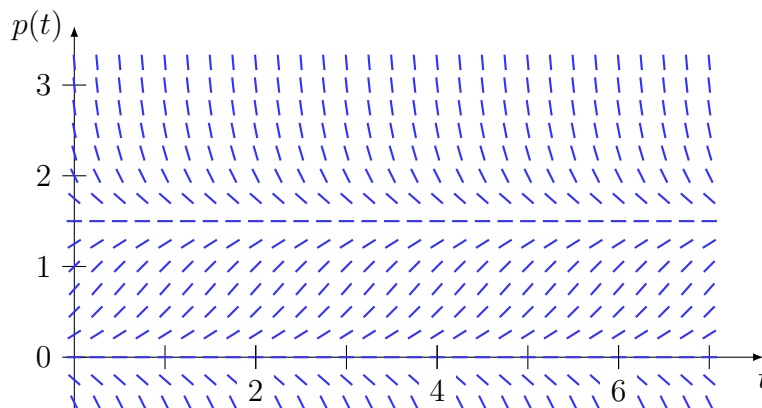
has a unique solution. If not, why not?

3. 10 pts. Determine whether the function $\theta(t) = 2e^{3t} - e^{2t}$ is a solution to the differential equation

$$\frac{d^2\theta}{dt^2} - \theta \frac{d\theta}{dt} + 3\theta = -2e^{2t}.$$

4. 10 pts. Determine for which values of m the function $\varphi(x) = e^{mx}$ is a solution to $2y''' + 9y'' - 5y' = 0$.

5. The logistic equation for the population p (in thousands) of a certain species is given by the equation $dp/dt = 3p - 2p^2$, which has the direction field below.



- (a) 4 pts. If the initial population is 3000 (i.e. $p(0) = 3$) what is the limiting population $\lim_{t \rightarrow \infty} p(t)$? Sketch the appropriate solution curve on the direction field.
- (b) 4 pts. If the initial population is 500 (i.e. $p(0) = 0.5$) what is the limiting population $\lim_{t \rightarrow \infty} p(t)$? Sketch the appropriate solution curve on the direction field.
- (c) 3 pts. Can a population of 2000 ever decline to 500?
6. 10 pts. Use Euler's Method with step size $h = 0.1$ to approximate the solution to the initial value problem $y' = x - y^2$, $y(1) = 0$, at the points $x = 1.1, 1.2, 1.3, 1.4, 1.5$.

7. 10 pts. Find the general solution to $\frac{dy}{dx} = \frac{e^{x+y}}{y-1}$.
8. 10 pts. Solve the initial value problem $\frac{1}{2}y' = \sqrt{y+1} \cos x$, $y(\pi) = 0$.
9. 10 pts. Find the general solution to $xy' + 3(y+x^2) = \frac{\sin x}{x}$.
10. 10 pts. Solve the initial value problem $t^2 \frac{dx}{dt} + 3tx = t^4 \ln t + 1$, $x(1) = 0$.
11. 10 pts. Solve the exact equation $(\cos x \cos y + 2x) - (\sin x \sin y + 2y)y' = 0$.

A couple trigonometric identities: $\sin(2\theta) = 2 \sin \theta \cos \theta$, $\cos(2\theta) = 2 \cos^2 \theta - 1$.