MATH 250
SPRING 2012
EXAM 4

NAME:

1. [5 pts.] Determine the Laplace transform of $f(t) = te^{3t}$.

2. [10 pts.] Use the Laplace transform table and linearity to determine $\mathcal{L}[t^2 - 3t - 3e^{-t}\sin 3t](s)$.

3. [10 pts.] Determine whether the function

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ t, & 2 \le t \le 10 \end{cases}$$

is piecewise-continuous on $[0, 10]$.

4. [15 pts.] Determine $\mathcal{L}[e^{7t}\sin^2 t](s)$.

5. [10 pts.] Determine $\mathcal{L}^{-1}[F](t)$, given that $F(s) = \dfrac{s+11}{(s-1)(s+3)}$.

6. [20 pts.] Solve the initial value problem $y'' + 2y' + 2y = t^2 + 4t$, $y(0) = 0$, $y'(0) = -1$, using the Method of Laplace Transforms.

7. The current $I(t)$ in a circuit is governed by the initial value problem $I''(t) + 2I'(t) + 2I(t) = g(t)$, $I(0) = 10$, $I'(0) = 0$, where

$$g(t) = \begin{cases} 20, & 0 \le t \le 3\pi \\ 0, & 3\pi < t \le 4\pi \\ 20, & t > 4\pi \end{cases}$$

   (a) [10 pts.] Express $g(t)$ in terms of window or step functions.

   (b) [10 pts.] Find $\mathcal{L}[g(t)](s)$.

   (c) [20 pts.] Solve the initial value problem by the Method of Laplace Transforms.

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$t^{-1/2}$	$\dfrac{\sqrt{\pi}}{\sqrt{s}}$	$s>0$
$t^{n-1/2},\ n=1,2,\ldots$	$\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$	$s>0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

$$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$

$$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$

$$\mathcal{L}[u(t-a)](s) = e^{-as}\mathcal{L}[1](s) = \frac{e^{-as}}{s}$$

$$\Pi_{a,b}(t) = u(t-a) - u(t-b) = \begin{cases} 0, & \text{if } t<a \\ 1, & \text{if } a\le t<b \\ 0, & \text{if } t\ge b \end{cases}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$