MATH 250
SPRING 2012
EXAM 1

NAME:

1. [5 pts. each] Write a differential equation that fits the physical description.

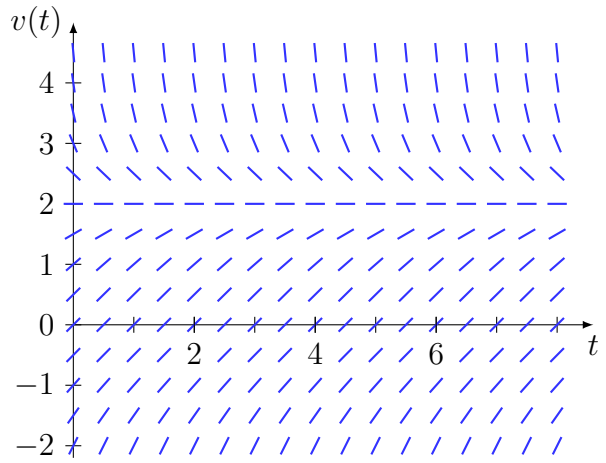
   (a) The velocity at time t of a particle moving along a straight line is proportional to the square of its position x.

   (b) The rate of change of the mass A of salt at time t is inversely proportional to the square root of the mass of salt present at time t.

2. [10 pts.] Show that $\varphi(x) = ce^{3x} + 1$ is a solution to $y' - 3y = -3$ for any choice of the constant c.

3. [10 pts.] Determine for which values of m the function $\varphi(x) = x^m$ is a solution to $x^2y'' - xy' - 5y = 0$.

4. [15 pts.] The velocity v at time t of an object falling in a viscous fluid is modeled by $v' = 1 - v^3/8$, with direction field given below. Sketch the solutions with initial conditions $v(0) = 0, 2, 4$. What velocity does the object approach as time increases? (This is known as the terminal velocity.)



5. [10 pts.] Use Euler's Method with step size $h = 0.1$ to approximate the solution to the initial value problem $y' = x - y^2$, $y(1) = 0$, at the points $x = 1.1, 1.2, 1.3, 1.4, 1.5$.

6. [10 pts.] Solve the initial value problem $y' = 8x^3e^{-2y}$, $y(1) = 0$.

7. [10 pts.] Find the general solution to $xy' + 3(y + x^2) = \dfrac{\sin x}{x}$.

8. [10 pts.] Solve the initial value problem $t^2\dfrac{dx}{dt} + 3tx = t^4 \ln t + 1$, $x(1) = 0$.

9. [10 pts.] Solve the exact equation $ye^{xy} - \dfrac{1}{y} + \left(xe^{xy} + \dfrac{x}{y^2}\right)y' = 0$.

A couple trigonometric identities: $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = 2\cos^2\theta - 1$.