MAT 250 Exam #3
Spring 2003

DO NOT WRITE ON THIS PAPER (except in the name box to the right). Show all work on blank paper provided. Points may be deducted for insufficient work even if correct answers are given.

Name:

Prob. Num.	Point Value	Points Given
1	7	
2a	7	
2b	7	
3a	7	
3b	7	
4	7	
5a	7	
5b	7	
6a	7	
6b	7	
6c	7	
7	7	
8	7	
9	9	
E	10	
Total	100	
Adj.		
Grade		

1) Solve the initial value problem: $y'' - 2y' + y = 0; \quad y(0) = 1, \; y'(0) = -2$

2) Use the Method of Undetermined Coefficients to find a general solution to the differential equation:

a. $y'' + 4y' - 2y = 2x^2 - 3x + 6$

b. $y'' - 5y' + 4y = 8e^x$

3) Use the Method of Undetermined Coefficients to find the <u>form</u> of a particular solution y_p for the differential equation, but do not bother to determine the coefficients:

a. $y'' - y = e^{2x} + xe^{2x} + x^2e^{2x}$

b. $y'' + 5y' + 6y = \sin x - \cos 2x$

4) A ¼-kg mass is attached to a spring with stiffness 8 N/m. The damping constant for the system is ¼ N-sec/m. If the mass is moved 1 m to the left of equilibrium and released, what is the maximum displacement to the right that it will attain?

5) Find the following Laplace transforms:

a. $\mathcal{L}\{e^{-2t}\cos\sqrt{3}t - t^2e^{-2t}\}$

b. $\mathcal{L}\{te^{3t}\sin 8t\}$

6) Find $\mathcal{L}^{-1}\{F\}$:

a. $F(s) = \dfrac{s+11}{(s-1)(s+3)}$

b. $F(s) = \ln\left(\dfrac{s-4}{s-3}\right)$

c. $F(s) = \dfrac{e^{-3s}}{s^2+9}$

7) Solve for $Y(s)$, the Laplace transform of the solution $y(t)$ to the initial value problem: $y'' + 3y = t^3; \quad y(0) = 0, \; y'(0) = 0.$

8) Express the given function using unit step functions and find its Laplace transform:

$$g(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$$

9) Solve the initial value problem using the method of Laplace transforms, then sketch the graph of the solution: $y'' + 5y' + 6y = t\,u(t-2); \quad y(0) = 0, \; y'(0) = 1.$ (Sketch is worth 2 points.)

<u>EXTRA CREDIT</u>:

A 2-kg mass is attached to a spring hanging from the ceiling, thereby causing the spring to stretch 20 cm upon coming to rest at equilibrium. At time $t = 0$ the mass is displaced 5 cm below the equilibrium position and released. At this same instant an external force $F(t) = 0.3\cos t$ N is applied to the system. If the damping constant for the system is 5 N-sec/m, determine the equation of motion for the mass. What is the resonance frequency of the system?