

**MAT 250 Exam #2
Spring 2003**

DO NOT WRITE ON THIS PAPER (except in the name box to the right). Show all work on blank paper provided. Points may be deducted for insufficient work even if correct answers are given.

Name:

Prob. Num.	Point Value	Points Given
1	10	
2	10	
3	10	
4	10	
5	10	
6a	3	
6b	3	
6c	3	
6d	3	
7	10	
8	10	
9	10	
10	10	
I	10	
II	10	
III	10	
Total	102	
Adj.		
Grade		

- 1) Show $(e^t x + 1)dt + (e^t - 1)dx = 0$ is exact, then solve given that $x(1) = 1$.
- 2) Use the method discussed under "Homogeneous Equations" to solve $(x^2 + y^2)dx + 2xy dy = 0$
- 3) Solve the equation $\frac{dy}{dx} = (2x + y - 1)^2$ using the method discussed under "Equations of the Form $dy/dx = G(ax+by)$ "
- 4) The air in a small room 12 ft by 8 ft by 8 ft is 3% carbon monoxide. Starting at $t = 0$, fresh air containing no carbon monoxide is blown into the room at a rate of 100 ft³/min. If air in the room flows out through a vent at the same rate, when will the air in the room be 0.01% carbon monoxide?
- 5) Determine whether the Existence & Uniqueness Theorem on page 165 applies to $x^2 z'' + xz' + z = \cos x$; $z(0) = 1$, $z'(0) = 0$. If it does, discuss what conclusions can be drawn. If it does not, explain why.
- 6) Let $y_1(x) = 2x^3$ and $y_2(x) = |x^3|$. Are y_1 and y_2 linearly independent on the following intervals? Use the definition for linear dependence of two functions (page 172) to explain why or why not. Find the appropriate constant multiple(s) for any case when they are linearly dependent.
 - a. $[0, \infty]$
 - b. $(-\infty, 0]$
 - c. $(-\infty, \infty)$
 - d. Compute the Wronskian $W[y_1, y_2](x)$ on the interval $(-\infty, \infty)$

- 7) Let operator L be defined by $L[y] \equiv (D^3 - x^2 D^2 + 4xD)[y]$, and compute $L[\ln x]$
- 8) Show that operator T, as defined by $T[y] \equiv y'' + (y'y^2)^{1/3}$, is a nonlinear operator.
- 9) Solve the second-order initial value problem:
 $y'' - 6y' + 9y = 0$; $y(0) = 2$, $y'(0) = \frac{25}{3}$.
- 10) Find a general solution to the third-order differential equation:
 $y''' + 3y'' - 4y' - 12y = 0$

EXTRA CREDIT (CHOOSE 1):

- I) One morning it began to snow very hard and continued snowing steadily throughout the day. A snowplow set out at 10:00 AM to clear a road, clearing 3 miles by 12:00 PM and an additional 1 mile by 2:00 PM. At what time did it start snowing?
- II) Problem # 4.4.14, all parts.
- III) Problem # 4.2.30, all parts