Math 250 Exam #5 Key (Spring 2016)

1 Employ partial fraction decomposition and linearity properties:

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{1}{(s-4)(s+5)}\right](t) &= \mathcal{L}^{-1}\left[\frac{1/9}{s-4} - \frac{1/9}{s+5}\right](t)\\ &= \frac{1}{9}\mathcal{L}^{-1}\left[\frac{1}{s-4}\right](t) - \frac{1}{9}\mathcal{L}^{-1}\left[\frac{1}{s+5}\right](t) = \frac{1}{9}e^{4t} - \frac{1}{9}e^{-5t}.\end{aligned}$$

2 Letting $Y = \mathcal{L}[y](s)$, from $\mathcal{L}[y'' - 4y' + 4y](s) = \mathcal{L}[t^2](s)$ we obtain

$$\big(s^2Y - sy(0) - y'(0)\big) - 4\big(sY - y(0)\big) + 4Y = \frac{2}{s^3},$$

and since $y(0) = 1$ and $y'(0) = 0$, it follows that

$$\begin{aligned}Y &= \frac{s-4}{s^2-4s+4} + \frac{2}{s^3(s^2-4s+4)} = \frac{s-4}{(s-2)^2} + \frac{2}{s^3(s-2)^2}\\ &= \frac{3/8}{s} + \frac{1/2}{s^2} + \frac{1/2}{s^3} + \frac{5/8}{s-2} - \frac{7/4}{(s-2)^2},\end{aligned}$$

and hence

$$y = \frac{3}{8} + \frac{1}{2}t + \frac{1}{4}t^2 + \frac{5}{8}e^{2t} - \frac{7}{4}te^{2t}.$$

3 First, note that $f(t) = 2 - 2u(t-1)$. Letting $Y = \mathcal{L}[y](s)$, from

$$\mathcal{L}[y'' + 4y](s) = \mathcal{L}[2 - 2u(t-1)](s)$$

we have

$$\big(s^2Y - sy(0) - y'(0)\big) + 4Y = \frac{2}{s} - \frac{2e^{-s}}{s}.$$

We're given that $y(0) = 0$ and $y'(0) = -1$, so

$$Y = \frac{2}{s(s^2+4)} - \frac{1}{s^2+4} - \frac{2e^{-s}}{s(s^2+4)},$$

and the rest is trivial.

4 Letting $Y = \mathcal{L}[y](s)$, from $\mathcal{L}[y' + y](s) = \mathcal{L}[\delta(t-3)](s)$ we obtain

$$\big(sY - y(0)\big) + Y = e^{-3s},$$

and since $y(0) = 2$ it follows that

$$Y = \frac{2}{s+1} + \frac{e^{-3s}}{s+1},$$

and hence

$$y = 2e^{-t} + u(t-3)e^{3-t}.$$