

1 $T'(t) = k[M(t) - T(t)]$, or simply $T' = k(M - T)$, where k is the constant of proportionality (not an arbitrary constant). Note: we could also write $T' = k(T - M)$, which in practice would result merely in the constant of proportionality k reversing sign.

2 We have

$$f(x, y) = 3x - \sqrt[3]{y - 1},$$

which is continuous everywhere. However

$$f_y(x, y) = -\frac{1}{3}(y - 1)^{-2/3} = -\frac{1}{3\sqrt[3]{(y - 1)^2}}$$

is not continuous on the line $y = 1$ since the function is not defined there. The initial point $(2, 1)$ lies on this line, and so the Existence-Uniqueness Theorem does not imply that the initial value problem has a unique solution.

3 Substitute $2e^{3t} - e^{2t}$ for θ in the equation to obtain

$$\begin{aligned} (2e^{3t} - e^{2t})'' - (2e^{3t} - e^{2t})(2e^{3t} - e^{2t})' + 3(2e^{3t} - e^{2t}) &= -2e^{2t} \\ (18e^{3t} - 4e^{2t}) - (2e^{3t} - e^{2t})(6e^{3t} - 2e^{2t}) + (6e^{3t} - 3e^{2t}) &= -2e^{2t} \\ 18e^{3t} - 4e^{2t} - 12e^{6t} + 10e^{5t} - 2e^{4t} + 6e^{3t} - 3e^{2t} &= -2e^{2t} \\ -12e^{6t} + 10e^{5t} - 2e^{4t} + 24e^{3t} - 7e^{2t} &= -2e^{2t} \end{aligned}$$

The last equation is not true for all t on *any* interval $I \subseteq \mathbb{R}$, and so the function $2e^{3t} - e^{2t}$ is *not* a solution to the ODE.

4 Substitute $\varphi(x) = e^{mx}$ for y to obtain

$$\begin{aligned} 2(e^{mx})''' + 9(e^{mx})'' - 5(e^{mx})' &= 0 \\ 2m^3e^{mx} + 9m^2e^{mx} - 5me^{mx} &= 0 \\ (2m^3 + 9m^2 - 5m)e^{mx} &= 0 \end{aligned}$$

To satisfy the equation for all x in some interval $I \subseteq \mathbb{R}$, it will be necessary to have

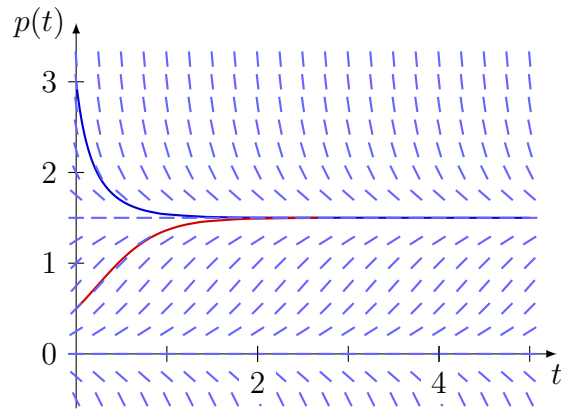
$$2m^3 + 9m^2 - 5m = 0.$$

Solving this equation for m , we have

$$m(2m - 1)(m + 5) = 0$$

and thus $m = 0, 1/2, -5$. This shows that $\varphi_1(x) = 1$, $\varphi_2(x) = e^{x/2}$, and $\varphi_3(x) = e^{-5x}$ are solutions to the ODE.

5 The solution curves corresponding to the initial conditions $p(0) = 3$ and $p(0) = 0.5$ are below. If $p(0) = 2$ we have $p(t) \rightarrow 1.5^+$ as $t \rightarrow \infty$, so a population of 2000 can never be 500.



6 We are given $(x_0, y_0) = (1, 0)$ and $h = 0.1$.

n	0	1	2	3	4	5
x_n	1.0	1.1	1.2	1.3	1.4	1.5
y_n	0.0000	0.1000	0.2090	0.3246	0.4441	0.5644

7 The equation is separable since

$$\frac{e^{x+y}}{y-1} = \frac{e^x e^y}{y-1} = e^x \cdot \frac{e^y}{y-1}.$$

Thus we have

$$\int \frac{y-1}{e^y} dy = \int e^x dx,$$

which yields

$$e^x + ye^{-y} = c$$

for arbitrary constant c .

8 Writing the equation as $y' = 2 \cos x \sqrt{y+1}$, we see the equation is separable. We get

$$\int \frac{1}{\sqrt{y+1}} dy = \int 2 \cos x dx.$$

This integrates easily to give

$$2\sqrt{y+1} = 2 \sin x + c.$$

Now, $y(\pi) = 0$ implies that

$$2\sqrt{0+1} = 2 \sin \pi + c,$$

or $c = 2$. The (implicit) solution to the IVP is thus $2\sqrt{y+1} = 2 \sin x + 2$, or $y = (\sin x + 1)^2 - 1$.

9 The equation may be written as

$$y' + \frac{3}{x}y = \frac{\sin x}{x^2} - 3x,$$

which is the standard form for a 1st-order linear ODE. An integrating factor is

$$\mu(x) = e^{\int 3/x dx} = e^{3 \ln x} = x^3.$$

Multiplying the ODE by x^3 gives

$$x^3 y' + 3x^2 y = x \sin x - 3x^4,$$

which becomes $(x^3 y)' = x \sin x - 3x^4$ and thus

$$x^3 y = \int x \sin x dx - \frac{3}{5}x^5 + c.$$

Integration by parts gives

$$\int x \sin x dx = \sin x - x \cos x,$$

so that $x^3 y = \sin x - x \cos x - \frac{3}{5}x^5 + c$ and therefore

$$y(x) = \frac{1}{x^3} \left(\sin x - x \cos x - \frac{3}{5}x^5 + c \right).$$

is the general solution.

10 The equation may be written as

$$x' + \frac{3}{t}x = t^2 \ln t + \frac{1}{t^2},$$

which is the standard form for a 1st-order linear ODE. An integrating factor is

$$\mu(x) = e^{\int 3/t dt} = t^3.$$

Multiplying the ODE by t^3 gives $t^3 x' + 3t^2 x = t^5 \ln t + t$, which becomes $(t^3 x)' = t^5 \ln t + t$ and thus

$$t^3 x = \int t^5 \ln t dt + \frac{1}{2}t^2 + c.$$

By integration by parts we find that

$$\int t^5 \ln t dt = \frac{1}{6}t^6 \ln t - \int \frac{1}{6}t^5 dt = \frac{t^6}{6} \ln t - \frac{t^6}{36},$$

and so we have

$$t^3 x = \frac{t^6}{6} \ln t - \frac{t^6}{36} + \frac{1}{2}t^2 + c.$$

Letting $t = 1$ and $x = 0$ (the initial condition) gives $0 = -\frac{1}{36} + \frac{1}{2} + c$, so that $c = -\frac{17}{36}$ and we obtain

$$x(t) = \frac{1}{6}t^3 \left(\ln t - \frac{1}{6} \right) + \frac{1}{2t} - \frac{17}{36t^3}$$

as the solution to the IVP.

11 Since the equation is exact there exists a function $F(x, y)$ such that

$$F_x(x, y) = \cos x \cos y + 2x \quad \text{and} \quad F_y(x, y) = -\sin x \sin y - 2y. \quad (1)$$

Integrate the first equation in (1) with respect to x to get

$$F(x, y) = \int (\cos x \cos y + 2x) dx + g(y) = \sin x \cos y + x^2 + g(y). \quad (2)$$

Differentiating this with respect to y yields

$$F_y(x, y) = -\sin x \sin y + g'(y),$$

and so using the second equation in (1) we obtain

$$-\sin x \sin y - 2y = -\sin x \sin y + g'(y),$$

or simply $g'(y) = -2y$. Hence $g(y) = -y^2 + c_1$ for some arbitrary constant c_1 , and so (2) becomes

$$F(x, y) = \sin x \cos y + x^2 - y^2 + c_1.$$

The general implicit solution to the ODE is therefore

$$\sin x \cos y + x^2 - y^2 + c_1 = c_2$$

for arbitrary c_2 , which we can write simply as

$$\sin x \cos y + x^2 - y^2 = c$$

by consolidating the arbitrary constants c_1 and c_2 .