MATH 250 EXAM #5 (FALL 2007)

1a $\dfrac{s+11}{(s-1)(s+3)} = \dfrac{A}{s-1} + \dfrac{B}{s+3} \Rightarrow s+11 = A(s+3) + B(s-1) \Rightarrow s+11 = (A+B)s + (3A-B) \Rightarrow$

$\begin{cases} A+B=1 \\ 3A-B=11 \end{cases} \Rightarrow A=3 \ \& \ B=-2$, so $\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\dfrac{3}{s-1} - \dfrac{2}{s+3}\right\} = \underline{\underline{3e^{t} - 2e^{-3t}}}$

1b $\dfrac{2s-1}{(s^2-4s+4)+2} = \dfrac{2s-1}{(s-2)^2+(\sqrt{2})^2} \Rightarrow \mathcal{L}^{-1}\{F(s)\} = 2\mathcal{L}^{-1}\left\{\dfrac{s-\frac{1}{2}}{(s-2)^2+2} = \dfrac{s-2}{(s-2)^2+2} + \dfrac{3/2}{(s-2)^2+2}\right\}$

$= 2\mathcal{L}^{-1}\left\{\dfrac{s-2}{(s-2)^2+(\sqrt{2})^2}\right\} + \dfrac{3}{\sqrt{2}}\mathcal{L}^{-1}\left\{\dfrac{\sqrt{2}}{(s-2)^2+(\sqrt{2})^2}\right\} = \underline{\underline{2e^{2t}\cos\sqrt{2}t + \dfrac{3}{\sqrt{2}}e^{2t}\sin\sqrt{2}t}}$

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2 $f(t) = -3 + 8u(t-4) + [-5+(t-4)]u(t-9) = -3 + 8u(t-4) + (t-9)u(t-9)$

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3 $\mathcal{L}\{y''\} + 9\mathcal{L}\{y\} = 10\mathcal{L}\{e^{2t}\} \Rightarrow s^2Y(s) - sy(0) - y'(0) + 9Y(s) = 10\left(\dfrac{1}{s-2}\right) \Rightarrow$

$s^2Y + s - 5 + 9Y = \dfrac{10}{s-2} \Rightarrow (s^2+9)Y = \dfrac{10}{s-2} + 5 - s \Rightarrow Y(s) = \dfrac{10}{(s-2)(s^2+9)} - \dfrac{s-5}{s^2+9}$

$= \dfrac{10/13}{s-2} - \dfrac{\frac{10}{13}s + \frac{20}{13}}{s^2+9} - \dfrac{s-5}{s^2+9} \Rightarrow y(t) = \dfrac{10}{13}\mathcal{L}^{-1}\left\{\dfrac{1}{s-2}\right\} - \dfrac{10}{13}\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+9}\right\} - \dfrac{20}{39}\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\}$

$- \mathcal{L}^{-1}\left\{\dfrac{s}{s^2+9}\right\} + \dfrac{5}{3}\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\} = \dfrac{10}{13}e^{2t} - \dfrac{10}{13}\cos 3t - \dfrac{20}{39}\sin 3t - \cos 3t + \dfrac{5}{3}\sin 3t \Rightarrow$

$\underline{\underline{y(t) = \dfrac{10}{13}e^{2t} - \dfrac{23}{13}\cos 3t + \dfrac{15}{13}\sin 3t}}$

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4 $\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = \mathcal{L}\{u(t-\pi)\} - \mathcal{L}\{u(t-2\pi)\} \Rightarrow$

$s^2Y - sy(0) - y'(0) + 4(sY - y(0)) + 4Y = \dfrac{e^{-\pi s}}{s} - \dfrac{e^{-2\pi}}{s} \Rightarrow$

$s^2Y + 4sY + 4Y = \dfrac{e^{-\pi s} - e^{-2\pi}}{s} \Rightarrow Y(s) = \dfrac{e^{-\pi s} - e^{-2\pi s}}{s(s^2+4s+4)} = \dfrac{e^{-\pi s} - e^{-2\pi s}}{s(s+2)^2} \Rightarrow$

$y(t) = \mathcal{L}^{-1}\left\{e^{-\pi s}\cdot\dfrac{1}{s(s+2)^2}\right\} - \mathcal{L}^{-1}\left\{e^{-2\pi s}\cdot\dfrac{1}{s(s+2)^2}\right\}$

$\dfrac{1}{s(s+2)^2} \to \dfrac{1/4}{s} - \dfrac{1/4}{s+2} - \dfrac{1/2}{(s+2)^2} \doteq F(s) \Rightarrow f(t) = \mathcal{L}^{-1}\{F(s)\} = \dfrac{1}{4} - \dfrac{1}{4}e^{-2t} - \dfrac{1}{2}e^{-2t}t$

$y(t) = f(t-\pi)u(t-\pi) - f(t-2\pi)u(t-2\pi) \Rightarrow$

$y(t) = \left[\dfrac{1}{4} - \dfrac{1}{4}e^{-2(t-\pi)} - \dfrac{1}{2}e^{-2(t-\pi)}(t-\pi)\right]u(t-\pi) - \left[\dfrac{1}{4} - \dfrac{1}{4}e^{-2(t-2\pi)} - \dfrac{1}{2}e^{-2(t-2\pi)}(t-2\pi)\right]u(t-2\pi) \Rightarrow$

$y(t) = \dfrac{1}{4}\left[1 - e^{-2t+2\pi} - 2(t-\pi)e^{-2t+2\pi}\right]u(t-\pi) - \dfrac{1}{4}\left[1 - e^{-2t+4\pi} - 2(t-2\pi)e^{-2t+4\pi}\right]u(t-2\pi)$