MATH 250 EXAM #4 KEY (FALL 2007)

$\boxed{1}$ • $0.75y'' + 0.6y' + 9y = 0$, $y(0) = -1.1$ m, $y'(0) = 0$

$y'' + 0.8y' + 12y = 0 \longrightarrow r^2 + 0.8r + 12 = 0 \rightarrow r = \dfrac{-0.8 \pm \sqrt{0.64 - 4(12)}}{2} \rightarrow \dfrac{-0.8 \pm i\sqrt{47.36}}{2}$

$\longrightarrow r \approx -0.4 \pm 3.44i$, so $y(t) = C_1 e^{-0.4t}\cos 3.44t + C_2 e^{-0.4t}\sin 3.44t$. Now...

$y(0) = -1.1 \Rightarrow -1.1 = C_1 \cdot 1 + C_2 \cdot 0 \Rightarrow C_1 = -1.1$.

Next, $y'(t) = e^{-0.4t}(-3.44 c_1 \sin 3.44t + 3.44 c_2 \cos 3.44t) - 0.4e^{-0.4t}(c_1 \cos 3.44t + c_2 \sin 3.44t)$,

so $y'(0) = 0 \Rightarrow 0 = 3.44c_2 - 0.4(c_1 + 0) \Rightarrow 3.44c_2 = 0.4c_1 \Rightarrow c_2 = \dfrac{0.4(-1.1)}{3.44} = -0.128$

Thus $y(t) = -1.1e^{-0.4t}\cos 3.44t - 0.128e^{-0.4t}\sin 3.44t$ is the equation of motion.

• Set $y'(t) = 0$ to get $y'(t) = -0.00032\cos 3.44t + 3.835\sin 3.44t = 0 \Rightarrow$

$3.835 \sin 3.44t = 0.00032\cos 3.44t \Rightarrow \tan 3.44t = 0.000083 \Rightarrow \tan 3.44t = 0$ (approx.)

$\Rightarrow 3.44t = \pi \Rightarrow t \approx 0.91$ sec. Maximum displacement is $y(0.91) = 0.76$ m.

$\boxed{2}$ $y_h'' + 2y_h = 0 \rightarrow r^2 + 2 = 0 \rightarrow r = \pm i\sqrt{2}$, so $y_h(t) = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t$ is the general homogeneous solution, where $y(0) = 0 \Rightarrow c_1 = 0 \Rightarrow y_h(t) = c_2 \sin\sqrt{2}t$. Now, $y_h'(t) = \sqrt{2}c_2 \cos\sqrt{2}t$, where $y'(0) = 0.5 \Rightarrow 0.5 = \sqrt{2}c_2 \cdot 1 \Rightarrow c_2 = \frac{1}{2\sqrt{2}}$. Hence $y_h(t) = \frac{1}{2\sqrt{2}}\sin\sqrt{2}t$.

For nonhomogeneous equation, let $y_p(t) = A\sin t + B\cos t$, so $y_p''(t) = -A\sin t - B\cos t$ and we have $y_p'' + 2y_p = 5\sin t \Rightarrow (-A\sin t - B\cos t) + 2(A\sin t + B\cos t) = 5\sin t \Rightarrow$

$A\sin t + B\cos t = 5\sin t \Rightarrow A = 5$ & $B = 0 \Rightarrow y_p(t) = 5\sin t$ is a particular solution.

General nonhomogeneous solution: $y(t) = \frac{1}{2\sqrt{2}}\sin\sqrt{2}t + 5\sin t$

$\boxed{3}$ $\mathcal{L}\{f\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^3 2te^{-st}\,dt + \int_3^\infty (0)\,dt = 2\int_0^3 te^{-st}\,dt$

Let $u = -st$, so $du = -s\,dt \Rightarrow dt = -\frac{1}{s}du$, and $t = -\frac{1}{s}u$. Now...

$\mathcal{L}\{f\} = 2\int_0^{-3s}\left(-\frac{1}{s}u\right)e^u \cdot \left(-\frac{1}{s}\right)du = \frac{2}{s^2}\int_0^{-3s} ue^u\,du = \frac{2}{s^2}\Big[(u-1)e^u\Big]_0^{-3s}$

$= \frac{2}{s^2}\left[(-3s-1)e^{-3s} - (0-1)e^0\right] = \frac{2}{s^2}\left[(-3s-1)e^{-3s} + 1\right] = \dfrac{(-6s-2)e^{-3s} + 2}{s^2}$

$\boxed{4}$ $\mathcal{L}\{e^{5t}\cos\sqrt{2}t\} - 8\mathcal{L}\{t^4\} = \dfrac{s-5}{(s-5)^2+2} - 8 \cdot \dfrac{4!}{s^{4+1}} = \dfrac{s-5}{(s-5)^2+2} - \dfrac{192}{s^5}$

$\boxed{5}$ $\sin 2x = 2\sin x \cos x \Rightarrow \sin x \cos x = \dfrac{\sin 2x}{2}$, so...

$\mathcal{L}\{\sin 4t \cos 4t\} = \mathcal{L}\left\{\dfrac{\sin 8t}{2}\right\} = \frac{1}{2}\mathcal{L}\{\sin 8t\} = \frac{1}{2} \cdot \dfrac{8}{s^2+64} = \dfrac{4}{s^2+64}$