MATH 250 EXAM #3 KEY (FALL 2007)

1 $\frac{dT}{dt} = k(T-86)$, where $T(t)$ is the temp. of the beer at time t; so $\frac{dT}{T-86} = k\,dt \Rightarrow$

$\ln(T-86) = kt + C \Rightarrow T-86 = e^{kt+C} \Rightarrow T(t) = 86 + Ke^{kt}$. From $T(0)=35$ & $T(5)=52$ we get: $35 = 86 + Ke^0 \Rightarrow K = -51$, so $T(t) = 86 - 51e^{kt}$; and $52 = 86 - 51e^{5k} \Rightarrow e^{5k} = \frac{2}{3} \Rightarrow 5k = \ln 2/3 \Rightarrow k \approx -0.081$, so $\underline{\underline{T(t) = 86 - 51e^{-0.081t}}}$

Now $T(20) = 86 - 51e^{-0.081(20)} = \underline{\underline{75.9\,°F}}$

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2 We suppose that $y(t) = e^{rt}$, which yields the auxiliary equation $r^2 + 2r - 1 = 0 \Rightarrow$

$r = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$; so general solution is $y(t) = c_1 e^{(-1-\sqrt{2})t} + c_2 e^{(-1+\sqrt{2})t}$

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3 By Method of Undetermined Coefficients $y_p(t) = Ae^{3t}$, so $y_p'(t) = 3Ae^{3t}$ & $y_p''(t) = 9Ae^{3t}$

Then equation becomes: $9Ae^{3t} + 2(3Ae^{3t}) - Ae^{3t} = e^{3t} \Rightarrow 14Ae^{3t} = e^{3t} \Rightarrow A = 1/14$.

Hence $y_p(t) = \frac{1}{14}e^{3t}$

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4 $y(t) = \frac{1}{14}e^{3t} + c_1 e^{(-1-\sqrt{2})t} + c_2 e^{(-1+\sqrt{2})t}$

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5 By Method of Undetermined Coefficients: we have $P_m(t) = 4t$, $\alpha = 1$, $\beta = 1$, $Q_n(t) = 0$.

Auxiliary eq. is $x^2 - 2x + 1 = 0$, so $x = 1$ and therefore $\alpha + i\beta = 1 + i$ is <u>not</u> a root. This means $s = 0$. Also $m = 1$ & $n = 0$, so $k = \max\{m,n\} = \max\{1,0\} = 1$. Now...

$$y_p(t) = t^0(A_1t + A_0)e^t \cos t + t^0(B_1t + B_0)e^t \sin t$$
$$= (A_1t + A_0)e^t \cos t + (B_1t + B_0)e^t \sin t$$