MATH 250 EXAM #4 KEY (F '06)

1 $y_p(t) = t^s A\cos 2t + t^s B\sin 2t$

Aux. Eq. is $x^2+4=0 \Rightarrow x=\pm 2i$

But $\alpha=0$ & $\beta=2$, so $\alpha+\beta i = 2i$ is a solution to aux. eq., meaning $s=1$.

So, $y_p(t) = At\cos 2t + Bt\sin 2t \Rightarrow 4y_p(t) = 4At\cos 2t + 4Bt\sin 2t$

Then $y_p'(t) = A\cos 2t - 2At\sin 2t + B\sin 2t + 2Bt\cos 2t$

And $y_p''(t) = -2A\sin 2t - 2A\sin 2t - 4At\cos 2t + 2B\cos 2t + 2B\cos 2t - 4Bt\sin 2t$

$= -4A\sin 2t + 4B\cos 2t - 4At\cos 2t - 4Bt\sin 2t$

So, $y''+4y = 8\sin 2t \Rightarrow -4A\sin 2t + 4B\cos 2t = 8\sin 2t \Rightarrow -4A=8$ & $4B=0 \Rightarrow$

$A=-2$ & $B=0$.

$\therefore\ y_p(t) = -2t\cos 2t$ ✓

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2 <u>Particular Solution:</u> $y_p(t) = t^s(A_2t^2+A_1t+A_0)$ (note: $r=0$, so e^{rt} is gone)

Aux. Eq.: $x^2-2x-3=0 \Rightarrow x=-1,3$, so $s=0$ (see part (a) of Method)

Hence $y_p(t) = A_2t^2+A_1t+A_0$

Then $y_p' = 2A_2t+A_1$ & $y_p'' = 2A_2$

So, $y''-2y'-3y = 3t^2-5 \Rightarrow$

$2A_2 - 2(2A_2t+A_1) - 3(A_2t^2+A_1t+A_0) = 3t^2-5 \Rightarrow$

$-3A_2t^2 + (-4A_2-3A_1)t + (2A_2-2A_1-3A_0) = 3t^2-5$, so...

$-3A_2=3$, $-4A_2-3A_1=0$, $2A_2-2A_1-3A_0=-5$

$A_2=-1$ $\quad A_1=4/3$ $\quad A_0=1/9$

$\therefore\ y_p(t) = -t^2+\frac{4}{3}t+\frac{1}{9}$

<u>Homogeneous Solution:</u> $y_h(t) = C_1e^{-t}+C_2e^{3t}$

<u>General Solution:</u> $y(t) = C_1e^{-t}+C_2e^{3t}-t^2+\frac{4}{3}t+\frac{1}{9}$ ✓

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3 For $y''-4y'+5y=e^{5t}$ we have $y_1(t) = t^sCe^{5t}$

Aux. eq. is $x^2-4x+5=0 \Rightarrow 5$ is not a solution $\Rightarrow s=0$, so $y_1(t)=Ce^{5t}$

For $y''-4y'+5y = t\sin 3t - \cos 3t$ we have $y_2(t) = t^s(A_1t+A_0)\cos 3t + t^s(B_1t+B_0)\sin 3t$

Now, $x^2-4x+5=0 \Rightarrow x=2\pm i$; but $\alpha+\beta i = 0+3i = 3i$, so $s=0$ again

So, $y_2(t) = (A_1t+A_0)\cos 3t + (B_1t+B_0)\sin 3t$.

By Superposition $y_p(t) = y_1(t)+y_2(t) = Ce^{5t} + (A_1t+A_0)\cos 3t + (B_1t+B_0)\sin 3t$ is the form of a particular solution to the given ODE (there are other forms)

$\boxed{4}$ $0.25y'' + 0.25y' + 8y = 0 \Rightarrow y'' + y' + 32y = 0$, where $y(0) = -1$, $y'(0) = 0$

Aux. Eq. is $x^2 + x + 32 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(32)}}{2} = \frac{-1 \pm \sqrt{-127}}{2} = -\frac{1}{2} \pm \frac{\sqrt{127}}{2} i$

So $y(t) = c_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{127}}{2} t\right) + c_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{127}}{2} t\right)$

Now, $y(0) = -1 \Rightarrow c_1 = -1$

Next, $y'(t) = e^{-\frac{1}{2}t}\left[-\frac{\sqrt{127}}{2} c_1 \sin\left(\frac{\sqrt{127}}{2} t\right) + \frac{\sqrt{127}}{2} c_2 \cos\left(\frac{\sqrt{127}}{2} t\right)\right] - \frac{1}{2} e^{-\frac{1}{2}t}\left[c_1 \cos\left(\frac{\sqrt{127}}{2} t\right) + c_2 \sin\left(\frac{\sqrt{127}}{2} t\right)\right]$

Now, $y'(0) = 0 \Rightarrow \frac{\sqrt{127}}{2} c_2 - \frac{1}{2} c_1 = 0 \Rightarrow \frac{\sqrt{127}}{2} c_2 + \frac{1}{2} = 0 \Rightarrow c_2 = -\frac{1}{\sqrt{127}}$

Then $y(t) = e^{-t/2}\left(-\cos \frac{\sqrt{127}}{2} t - \frac{1}{\sqrt{127}} \sin \frac{\sqrt{127}}{2} t\right)$

Maximum displacement time? Solve $y'(t) = 0 \Rightarrow \sin\left(\frac{\sqrt{127}}{2} t\right) = 0 \Rightarrow \frac{\sqrt{127}}{2} t = \pi$

(note: π is the 1st positive value possible for $\frac{\sqrt{127}}{2} t$)

So $t = \frac{2\pi}{\sqrt{127}} \approx 0.5575$ second.

So, max. displacement $= y(0.5575) = 0.7567$ meter