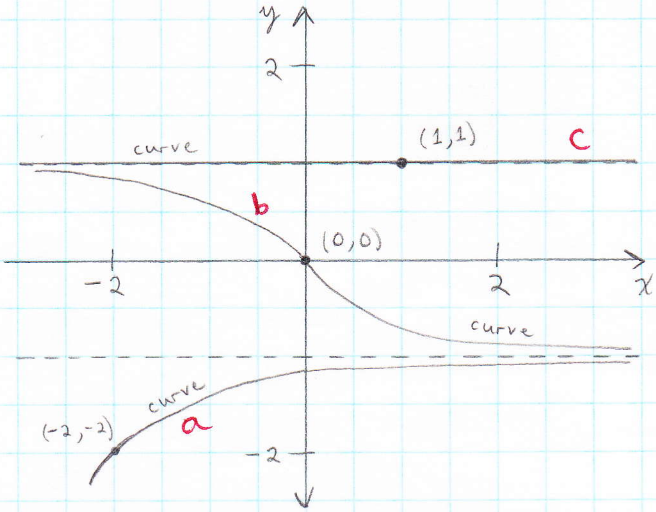
MATH 250 EXAM #1 KEY (FALL 2004)

1a

1b

1c



1d

Part (a): As $x \to \infty$, $y \to -1$ from below

As $x \to -\infty$, $y \to -\infty$

Part (b): As $x \to \infty$, $y \to -1$ from above

As $x \to -\infty$, $y \to 1$ from below

Part (c): As $x \to \infty$, $y \equiv 1$

As $x \to -\infty$, $y \equiv 1$

2

$\frac{1}{y}dy = \sin x\,dx \Rightarrow \ln|y| = -\cos x + C$

Using $y(\pi) = -3$:

$\ln|-3| = -\cos(\pi) + C \Rightarrow \ln 3 = 1 + C \Rightarrow$

$C = \ln 3 - 1$

Thus $\ln|y| = -\cos x + \ln 3 - 1$ ✓

Or: $|y| = e^{-\cos x + C} = Ke^{-\cos x} \Rightarrow$

$y = \pm Ke^{-\cos x} \Rightarrow y = Ae^{-\cos x}$,

where $A = \pm K$

Then $y(\pi) = -3$ yields $-3 = Ae^{-\cos \pi} \Rightarrow$

$-3 = Ae^{1} \Rightarrow A = -3e^{-1}$

Then $y = -3e^{-1}e^{-\cos x} \Rightarrow$

$y = -3e^{-\cos x - 1}$ ✓

3

$\frac{dy}{dx} + \frac{2}{x}y = 5x^2$

Let $\mu(x) = e^{\int 2/x\,dx} = e^{2\ln x} = x^2$

We get: $x^2y' + 2xy = 5x^4 \Rightarrow$

$(x^2y)' = 5x^4 \Rightarrow x^2y = \int 5x^4dx + C \Rightarrow$

$x^2y = x^5 + C \Rightarrow$

$y = x^3 + Cx^{-2}$ ✓

4

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(\cos x \cos y + 2x) = -\cos x \sin y$

$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(-\sin x \sin y - 2y) = -\cos x \sin y$

So $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ exactness.

Find F such that $\frac{\partial F}{\partial x} = M$ & $\frac{\partial F}{\partial y} = N$

So $F(x,y) = \int M(x,y)\,dx + g(y) \Rightarrow$

$F(x,y) = \int(\cos x \cos y + 2x)dx + g(y)$

$= \sin x \cos y + x^2 + g(y)$

Then: $N(x,y) = \frac{\partial F}{\partial y} = \frac{\partial}{\partial y}(\sin x \cos y + x^2 + g(y)) \Rightarrow$

$N(x,y) = -\sin x \sin y + g'(y) \Rightarrow$

$-\sin x \sin y - 2y = -\sin x \sin y + g'(y) \Rightarrow$

$g'(y) = -2y$

So $g(y) = -y^2 + B$

Then $F(x,y) = \sin x \cos y + x^2 - y^2 + B$

So $\sin x \cos y + x^2 - y^2 = C$ ✓

5

We need compatibility to have exactness:

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Multiply the diff. eq. by $x^m y^n$:

$$\underbrace{(12x^m y^n + 5x^{m+1}y^{n+1})}_{M(x,y)}dx + \underbrace{(6x^{m+1}y^{n-1} + 3x^{m+2}y^n)}_{N(x,y)}dy = 0$$

$\frac{\partial M}{\partial y} = 12n x^m y^{n-1} + 5(n+1)x^{m+1}y^n$

$\frac{\partial N}{\partial x} = 6(m+1)x^m y^{n-1} + 3(m+2)x^{m+1}y^n$

We need $12n = 6(m+1)$ & $5(n+1) = 3(m+2)$

So: $\begin{cases} 12n - 6m = 6, & (A) \\ 5n - 3m = 1, & (B) \end{cases}$

(A) yields $2n - m = 1 \Rightarrow m = 2n - 1$

Putting this into (B) yields:

$5n - 3(2n-1) = 1 \Rightarrow 5n - 6n + 3 = 1 \Rightarrow$

$-n + 3 = 1 \Rightarrow n = 2$ & $m = 3$

So $x^m y^n = x^3 y^2$ is our integrating factor,

giving us $(12x^3y^2 + 5x^4y^3)dx + (6x^4y + 3x^5y^2)dy = 0$

Solve as in #4...

$F(x,y) = \int (12x^3y^2 + 5x^4y^3)\,dx + g(y) = 3x^4y^2 + x^5y^3 + g(y)$

Then $6x^4y + 3x^5y^2 = 6x^4y + 3x^5y^2 + g'(y) \Rightarrow g'(y) = 0 \Rightarrow g(y) = A$

Thus $F(x,y) = 3x^4y^2 + x^5y^3 + A$

Solution: $F(x,y) = C \Rightarrow 3x^4y^2 + x^5y^3 + A = C \Rightarrow \underline{\underline{3x^4y^2 + x^5y^3 = B}}$ (where $B = C - A$)

$\boxed{6a}$ Ordinary, 4th-order, nonlinear equation with y dependent & x independent

$\boxed{6b}$ Ordinary, 3rd-order, nonlinear equation with y dependent & x independent

$\boxed{6c}$ Partial, 2nd-order equation with u dependent & x, t independent