

1. 20 pts. Use the Method of Frobenius to get two linearly independent series solutions about the regular singular point $x = 0$ for

$$2xy'' + 5y' + xy = 0,$$

then form the general solution on $(0, \infty)$.

2. 20 pts. Solve the initial-value problem

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where

$$f(t) = \begin{cases} t, & 0 \leq t < \pi/2 \\ \pi/2, & t \geq \pi/2 \end{cases}$$

3. 10 pts. Solve the integro-differential equation

$$y'(t) = \cos t + \int_0^t y(\tau) \cos(t - \tau) d\tau, \quad y(0) = 1.$$

4. 20 pts. Use the Laplace transform to solve the initial-value problem

$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 0, \quad y'(0) = 2.$$

$f(t)$	$\mathcal{L}[f](s)$	$\text{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\frac{2bs}{(s^2 + b^2)^2}$	$s > 0$
$t \cos bt$	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\frac{b}{(s - a)^2 + b^2}$	$s > a$
$e^{at} \cos bt$	$\frac{s - a}{(s - a)^2 + b^2}$	$s > a$
$e^{at} t^n, n = 0, 1, \dots$	$\frac{n!}{(s - a)^{n+1}}$	$s > a$
$u(t - a), a \geq 0$	$\frac{e^{-as}}{s}$	$s > 0$
$\delta(t - a), a \geq 0$	e^{-as}	$s > 0$
$(f * g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s > 0$

$$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$$

$$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$$

$$\mathcal{L}[e^{at} f(t)](s) = \mathcal{L}[f(t)](s - a)$$

$$\mathcal{L}[f(t - a)u(t - a)](s) = e^{-as}\mathcal{L}[f(t)](s)$$

$$\mathcal{L}[g(t)u(t - a)](s) = e^{-as}\mathcal{L}[g(t + a)](s)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$