MATH 250
FALL 2021
EXAM 4

NAME:

1. $\boxed{\text{20 pts.}}$ A force of 400 newtons stretches a spring 2 meters. A mass of 50 kg is attached to the end of the spring and is initially released from the equilibrium position with an upward velocity of 10 m/s. Find the equation of motion, and put it in the form $x(t) = A\sin(\omega t + \varphi)$. What is the period of motion? At what times is the mass heading downward at a velocity of 5 m/s?

2. $\boxed{\text{20 pts.}}$ Find two linearly independent power series solutions to

$$y'' + xy' + 2y = 0$$

about the ordinary point $x = 0$. Determine at least the first four nonzero terms of each series.

3. $\boxed{\text{10 pts.}}$ Use the Laplace transform to solve the initial-value problem

$$\frac{dy}{dt} + 3y = 2, \quad y(0) = -3.$$

4. $\boxed{\text{20 pts.}}$ Use the Laplace transform to solve the initial-value problem

$$y'' - 2y' + y = e^t, \quad y(0) = 0, \quad y'(0) = 5.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t \sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s > 0$
$t \cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s > 0$
$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s > a$
$e^{at}t^n,\ n = 0, 1, \ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s > a$
$u(t-a),\ a \geq 0$	$\dfrac{e^{-as}}{s}$	$s > 0$
$\delta(t-a),\ a \geq 0$	e^{-as}	$s > 0$
$(f * g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s > 0$

$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

$\mathcal{L}[e^{at}f(t)](s) = \mathcal{L}[f(t)](s-a)$

$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$